A recursive group decision making procedure for choosing qualified individuals

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Abstract

In this paper we introduce a new group decision procedure that follows a recursive pattern. In the first stage, the members of a group show their opinions on all the individuals of that group, regarding a specific attribute, by means of assessments within the unit interval. Taking into account this information, some aggregation operators and a family of thresholds, a subgroup of individuals is selected: those members whose collective assessment reach a specific threshold. Now only the opinions of this qualified subgroup are taken into account and a new subgroup emerges in the implementation of the aggregation phase. We analyze when this recursive procedure converges providing a final subgroup of qualified members, taking into account some extended OWAs.

Keywords: Group decision making; OWA operators; aggregation; qualification.

1 Introduction

In some occasions a group of individuals has to select a subgroup for doing a task or regarding an attribute. This is the case of a group of experts that has to decide which members should participate in a concrete task. Another possibility consists on determining which members of a society are the best for constituting a committee. There exist in the literature some works where the problem arises in choosing the members of the society satisfying a social identity (see Kasher and Rubinstein [9]), or with respect to a general attribute (see Samet and Schmeidler [11]). We note that in these mentioned papers, only dichotomous assessments are allowed.

Aggregation operators allow us to generate a collective assessment to each individual of the group taking into account the individual opinions (see Fodor and Roubens [5], Grabisch, Orlovski and Yager [6] and Calvo, Kolesárova, Komorníková and Mesiar [3], among others). A simple way of selecting a subgroup can be done by means of thresholds or quota: all the members reaching the quota have to be chosen.

Typically, those members enrolled in this conformed category have more valuable opinions about the task or topic under discussion. It seems therefore reasonable to choose a *stable group*, i.e., a group such that when considering their assessments on all society members, they would select themselves, and only themselves, for the task. To analyze the possibility of arriving to stable subgroups, in this paper we consider a sequential procedure where in each stage a subgroup is selected, and taking the opinions of this subgroup a new subgroup emerges. We analyze when this procedure converges to a final set of selected individuals. For extra analysis of this procedure, see Ballester and García-Lapresta [1, 2].

Among the large variety of aggregation operators, in this paper we have considered OWA ("Ordered Weighted Averaging") operators (Yager [12]); a wide study can be found in Yager and Kacprzyk [13]. See Fodor and Roubens [5, 5.8] and Calvo, Kolesárova, Komorníková and Mesiar [3, 4.2] as well.

The paper is organized as follows. In Section 2 we introduce the framework for developing the group decision procedure we analyze in the paper. Section 3 includes some important families of collective evaluation mechanisms suitable for the recursive decision problem. In Section 4 we show the results of the paper. Section 5 shows some illustrative examples and Section 6 contains some conclusions.

2 Preliminaries

Consider a finite set of individuals $N = \{1, 2, \ldots, n\}$ with $n \geq 2$. We use 2^N to denote the power set of N, i.e., the set of all the subsets of N, and $|S|$ is the cardinal of S. A *profile* is an $n \times n$ matrix

$$P = \begin{pmatrix} p_{11} & \cdots & p_{1j} & \cdots & p_{1n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ p_{i1} & \cdots & p_{ij} & \cdots & p_{in} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ p_{n1} & \cdots & p_{nj} & \cdots & p_{nn} \end{pmatrix}$$

with values in the unit interval, where p_{ij} is the assessment with which the individual i evaluates individual j as being qualified to belong to the committee in question. The set of profiles is denoted by $\mathcal{P}$. Given a subset of individuals $S \subseteq N$, P_S denotes the $|S| \times n$ submatrix of P composed by those $i-$rows with $i \in S$. Given $j \in N$, we denote by P_S^j the j-th column vector of P_S. Finally, we denote by $\sigma(\, P_S^j\,)$ a permutation of vector P_S^j such that opinions are ordered from best to worst; and $[\sigma(\, P_S^j\,)]_i$ denotes the i-th component of $\sigma(\, P_S^j\,)$.

Definition 1 *A* Committees' Evaluation Mechanism (CEM) *is a family of functions* $\{v_S\}$, *with* $\emptyset \neq S \subseteq N$, *where* $v_S : \mathcal{P} \times N \longrightarrow [0, 1]$ *is a function that, given* $P \in \mathcal{P}$, *assigns a collective assessment* $v_S(P, j) \in [0, 1]$ *to each individual* $j \in N$ *in such a way that* $v_S(P, j) = v_S(Q, j)$ *for all* $P, Q \in \mathcal{P}$ *satisfying* $P_S^j = Q_S^j$.

According to the previous definition, the collective assessment that the subgroup S provides to individual j, $v_S(P, j)$, only depends on the individual assessments of S on the individual j.

A CEM determines individual qualification only in gradual terms. A very natural way to convert a gradual opinion into a dichotomic assessment is by means of thresholds or quotas. An individual is qualified if the collective assessment is above a fixed quota.

Definition 2 *A family of values* $\{\alpha_S\}$, $\emptyset \neq S \subseteq N$, *and* $\alpha_S \in (0, 1]$ *for every* S, *is called a* Threshold Mechanism (TM).

Given a CEM $\{v_S\}$ *and a TM* $\{\alpha_S\}$*, the family of functions* $\{V_S\}$*, with* $S \subseteq N$*, where* $V_S : \mathcal{P} \longrightarrow 2^N$ *is the function that, given* $P \in \mathcal{P}$*, qualifies a new subgroup* $V_S(P)$ *as follows:*

$$V_S(P) = \begin{cases} \{j \in N \mid v_S(P, j) \geq \alpha_S\}, & \text{if } S \neq \emptyset, \\ \emptyset, & \text{otherwise.} \end{cases}$$

The family of functions $\{V_S\}$ *is called the* Committees' Qualification Mechanism (CQM) *associated with* $\{\alpha_S\}$ *and* $\{v_S\}$*.*

Definition 3 *Given a CQM* $\{V_S\}$*, the sequence* $\{S_t\}$*, where* $S_1 = N$ *and* $S_{t+1} = V_{S_t}(P)$*, is called* committees' sequence*. A committees' sequence is said to be* convergent *if* $\{S_t\}$ *has a limit* $\lim S_t$ *(and it is also said that the sequence* converges *to* $\lim S_t$*). The CQM is said to be* convergent *if for any* $P \in \mathcal{P}$ *the committees' sequence generated is convergent.*

Notice that, since society is a finite set, chain convergence can also be expressed as: there exists a positive integer q such that $S_t = S_q$ for every $t \geq q$.

Figure 1 shows in a nutshell the main components of a sequential process of evaluation.

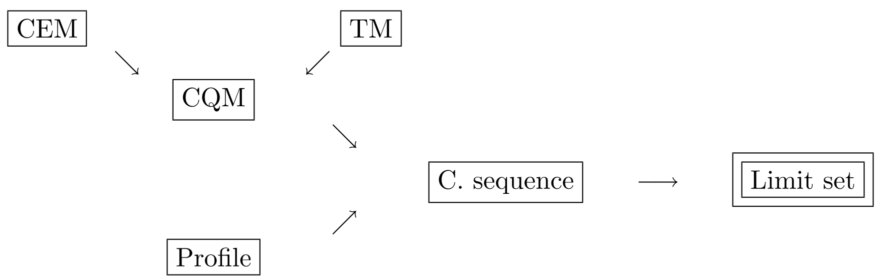

Figure 1. Iterative procedure.

3 Describing families of CEMs

In this section, we focus our attention on how general aggregations operators and classes of extended OWAs can be used in the recursive procedure introduced in the previous section.

3.1 The use of aggregation operators to construct CEMs

In some occasions, aggregation operators are defined for a given number of individual opinions. Notice however that for our model to apply, we have to consider collections of operators, one for each possible subgroup of individuals in society. There are multiple options to consider here, and it is our purpose to analyze general classes of operators. For the sake of a more intuitive analysis, we adopt here the following two assumptions about CEMs to be used:

SE *Self-Exclusion*: When an individual had to decide on herself, she will exclude her opinion when possible (i.e., she is not the only reviewer).

CS *Cardinality-Symmetry*: When the same number of opinions have to be aggregated, the same operator will be applied (independently of the names of the reviewers).

Both of them could be eliminated without altering significatively the results, but we consider this model more natural.

3.2 OWAs

A broad family of operators that makes sense in an aggregation process is that of OWAs.

Definition 4 *The* OWA operator *of dimension p associated with the weighting vector $(w_1^p, \dots, w_p^p) \in [0,1]^p$ such that $w_1^p + \cdots + w_p^p = 1$ is the function $w^p : [0,1]^p \longrightarrow [0,1]$ defined by*

$$w^p(a_1, \dots, a_p) = w_1^p \cdot b_1 + \cdots + w_p^p \cdot b_p,$$

with b_i the i-th largest of the $a_1, \dots, a_p$.

The notion of extended aggregation functions and extended OWAs (see Mayor and Calvo [10] and Calvo and Mayor [4]) becomes useful for our purposes.

Definition 5 *An* Extended OWA (EOWA) *operator is a family of OWA operators $\{w^p\}$ where w^p is an OWA of dimension p with associated weights $(w_1^p, \dots, w_p^p) \in [0,1]^p$ such that $w_1^p + \cdots + w_p^p = 1$.*

We now formally explicit how an EOWA can be considered a CEM following the assumptions **SE** and **CS**. This can be done by considering, for every $\emptyset \neq S \subseteq N$:

1. The OWA of dimension $|S|$, $w^{|S|}$, applied to P_S^j, if $j \notin S$.
2. The OWA of dimension $|S|-1$, $w^{|S|-1}$, applied to $P_{S\setminus\{j\}}^j$, if $j \in S \neq \{j\}$.
3. The OWA of dimension 1, w^1, i.e., the identity, applied to $P_{\{j\}}^j = p_{jj}$, if $S = \{j\}$.

It could be seem reasonable to impose certain conditions about the consistency of the threshold values and EOWAs across committees. However, we initially avoid such conditions in order to present the most general results. As it will be shown in Section 4, convergence is close related to the chosen families and therefore, the imposition of consistency conditions could help to get new insights for our recurrent model.

The following examples explicit relevant families of EOWAs.

Example 1 The *maximum* or *best* aggregator can be defined, for any p, through the weights

$$(w_1^p, \dots, w_p^p) = (1, 0, \dots, 0, 0).$$

The associated CEM $\{b_S\}$ is described in Table 1.

Table 1: CEM associated with the *best* aggregator

Cases	CEM: $b_S(P,j)$
(1) $j \notin S$	$\max\{p_{ij} \mid i \in S\}$
(2) $j \in S \neq \{j\}$	$\max\{p_{ij} \mid i \in S \setminus \{j\}\}$
(3) $S = \{j\}$	p_{jj}

The associated CQM $\{B_S\}$ is defined by

$$j \in B_S(P) \Leftrightarrow b_S(P,j) \geq \alpha_S.$$

The *minimum* or *worst* aggregator has as associated weights: $(w_1^p, \ldots, w_p^p) = (0, 0, \ldots, 0, 1)$. The associated CEM and CQM are denoted by $\{w_S\}$ and $\{W_S\}$, respectively.

The *median* aggregator has weights

- $w^p_{\frac{p+1}{2}} = 1$ and $w_i^p = 0$ otherwise, whenever p is odd.
- $w^p_{\frac{p}{2}} = w^p_{\frac{p}{2}+1} = 0.5$ and $w_i^p = 0$ otherwise, whenever p is even.

The associated CEM and CQM are denoted by $\{m_S\}$ and $\{M_S\}$, respectively.

The description of the associated CEMs $\{w_S\}$ and $\{m_S\}$, and the CQMs $\{W_S\}$ and $\{M_S\}$, follows a similar pattern to that of the maximum in Table 1 and thus we omit them.

Example 2 The *average* aggregator can be defined, for any p through the weights

$$(w_1^p, \ldots, w_p^p) = \left(\tfrac{1}{p}, \ldots, \tfrac{1}{p}\right).$$

The associated CEM $\{a_S\}$ is described in Table 2.

The associated CQM $\{A_S\}$ is defined by

$$j \in A_S(P) \Leftrightarrow a_S(P,j) \geq \alpha_S.$$

Example 3 The *olympic* aggregator can be defined, when $p \geq 3$, through the weights $w_1^p = w_p^p = 0$ and $w_i^p = \frac{1}{p-2}$ otherwise. For the cases $p = 1, 2$ we use as a natural extension the average aggregator. The associated CEM $\{o_S\}$ is described in Table 3.

The associated CQM $\{O_S\}$ is defined by

$$j \in O_S(P) \Leftrightarrow o_S(P,j) \geq \alpha_S.$$

Table 2: CEM associated with the *average* aggregator

Cases	CEM: $a_S(P,j)$
(1) $j \notin S$	$\frac{1}{\lvert S\rvert}\sum_{i\in S} p_{ij}$
(2) $j \in S \neq \{j\}$	$\frac{1}{\lvert S\rvert - 1}\sum_{i\in S\setminus\{j\}} p_{ij}$
(3) $S = \{j\}$	p_{jj}

4 The results

In this section we provide some necessary and sufficient conditions for the convergence of CQMs based on EOWAs. We pay special attention to CQMs based on examples provided in Section 3.

In our first result, we present a very soft necessary condition for the convergence of the CQM derived from the use of EOWAs.

Proposition 1 *Let $\{V_S\}$ be a CQM associated with an EOWA $\{w^p\}$ and a TM $\{\alpha_S\}$. If there exists a dimension $u \leq n-1$ such that $w_1^u = 0$, then $\{V_S\}$ is not convergent.*

PROOF: Suppose there exists a dimension $u \leq n-1$ such that $w_1^u = 0$. Consider a subgroup of the society U such that $|U| = u$, where it is clear that $u > 1$. Let $U = \{i_1, \ldots, i_u\}$ and the profile P defined by:

$$p_{ij} = \begin{cases} 1, & \text{if } (i = i_1 \text{ and } j \in U) \text{ or } j = i_1, \\ 0, & \text{otherwise.} \end{cases}$$

Depending on the value of α_N and the EOWA $\{w^n\}$, it will be either $S_2 = \{i_1\}$ or $S_2 = U$. It is straightforward to see that $V_{\{i_1\}}(P) = U$. Whenever the committee is U, the opinion of i_1 is considered with a weight of $w_i^u = 0$ and therefore, $V_U(P) = \{i_1\}$. Thus, the CQM $\{V_S\}$ is not convergent. ■

The previous result allows us to discuss the instability of a broad family of OWAs.

Corollary 1 *If $n \geq 4$, given any TM $\{\alpha_S\}$, the CQMs $\{W_S\}$, $\{M_S\}$ and $\{O_S\}$ are not convergent. If $n \geq 3$, the result still holds for $\{W_S\}$.*

PROOF: All the presented EOWAs have $w_1^3 = 0$. Given the fact that $n \geq 4$ and Proposition 1, the result is straightforward. ■

We now provide sufficient conditions for ensuring the convergence of CQMs based on EOWAs.

Table 3: CEM associated with the *olympic* aggregator

Cases	CEM: $o_S(P,j)$
(1a) $j \notin S$ and $\|S\| > 2$	$\frac{1}{\|S\|-2}\Big(\sum_{i\in S} p_{ij} - \min\{p_{ij} \mid i \in S\} - \max\{p_{ij} \mid i \in S\}\Big)$
(1b) $j \notin S$ and $\|S\| \leq 2$	$\frac{1}{\|S\|}\sum_{i\in S} p_{ij}$
(2a) $j \in S$ and $\|S\| > 3$	$\frac{1}{\|S\|-3}\Big(\sum_{i\in S\setminus\{j\}} p_{ij} - \min\{p_{ij} \mid i \in S \setminus \{j\}\} - \max\{p_{ij} \mid i \in S \setminus \{j\}\}\Big)$
(2b) $j \in S$ and $\|S\| \in \{2,3\}$	$\frac{1}{\|S\|-1}\sum_{i\in S\setminus\{j\}} p_{ij}$
(3) $S = \{j\}$	p_{jj}

Proposition 2 *Let* $\{V_S\}$ *be a CQM associated with an EOWA* $\{w^p\}$ *and a TM* $\{\alpha_S\}$*. If the following two conditions are satisfied:*

1. *For all positive integers* m, u, v *such that* $m \leq u \leq v$ *it holds*

$$\sum_{i=1}^{m} w_i^u \geq \frac{u}{v}\sum_{i=1}^{m} w_i^v,$$

2. *For all non-empty sets* $U, V \subseteq N$ *such that* $U \subseteq V$ *it holds*

$$|U|\,\alpha_U \geq (|V|-1)\,\alpha_V,$$

then $\{V_S\}$ *is convergent.*

Proof: Suppose conditions (1) and (2) hold. Given any profile P, we prove by induction that the committee sequence is decreasing, hence, convergent. Since $S_1 = N$, obviously $S_2 \subseteq S_1$. Suppose $S_{k+1} \subseteq S_k$ is true for $k = 1, \ldots, t-1$ (in particular $S_t \subseteq S_{t-1}$). In order to prove $S_{t+1} \subseteq S_t$ by way of contradiction, suppose there exists an individual $j \in S_{t+1}$ such that $j \notin S_t$. Consider the greatest integer m such that $j \in S_m$, with $m \leq t-1$. Notice that this is well-defined, since $S_1 = N$. By the induction hypothesis, it must be that

$S_t \subseteq S_m$ and $S_t = S_t \setminus \{j\} \subseteq S_m \setminus \{j\}$. Therefore, the vector composed by the $|S_t|$ greatest values in $\{p_{ij}\}_{i \in S_m \setminus \{j\}}$ dominates (component by component) the vector composed by the $|S_t|$ greatest values in $\{p_{ij}\}_{i \in S_t}$. Hence

$$\begin{aligned}
& w^{|S_m|-1} \cdot \sigma\left(P^j_{S_m \setminus \{j\}}\right) = \\
& \left(w_1^{|S_m|-1}, \ldots, w_{|S_m|-1}^{|S_m|-1}\right) \cdot \left(\left[\sigma\left(P^j_{S_m \setminus \{j\}}\right)\right]_1, \ldots, \left[\sigma\left(P^j_{S_m \setminus \{j\}}\right)\right]_{|S_m|-1}\right) \geq \\
& \left(w_1^{|S_m|-1}, \ldots, w_{|S_t|}^{|S_m|-1}\right) \cdot \left(\left[\sigma\left(P^j_{S_m \setminus \{j\}}\right)\right]_1, \ldots, \left[\sigma\left(P^j_{S_m \setminus \{j\}}\right)\right]_{|S_t|}\right) \geq \\
& \left(w_1^{|S_m|-1}, \ldots, w_{|S_t|}^{|S_m|-1}\right) \cdot \sigma\left(P^j_{S_t}\right).
\end{aligned}$$

Condition (1) guarantees

$$\left(w_1^{|S_m|-1}, \ldots, w_{|S_t|}^{|S_m|-1}\right) \cdot \sigma\left(P^j_{S_t}\right) \geq w^{|S_t|} \cdot \sigma\left(P^j_{S_t}\right) \frac{|S_t|}{|S_m| - 1}.$$

By the fact that $j \in S_{t+1} = V_{S_t}(P)$, it must be

$$w^{|S_t|} \cdot \sigma\left(P^j_{S_t}\right) \geq \alpha_{S_t},$$

i.e.,

$$w^{|S_t|} \cdot \sigma\left(P^j_{S_t}\right) \frac{|S_t|}{|S_m| - 1} \geq \alpha_{S_t} \frac{|S_t|}{|S_m| - 1}.$$

Since $S_t \subseteq S_m$, condition (2) ensures

$$|S_t| \, \alpha_{S_t} \geq (\, |S_m| - 1) \, \alpha_{S_m},$$

i.e.,

$$\alpha_{S_t} \frac{|S_t|}{|S_m| - 1} \geq \alpha_{S_m}.$$

Hence, we obtain

$$w^{|S_m|-1} \cdot \sigma\left(P^j_{S_m \setminus \{j\}}\right) \geq \alpha_{S_m}.$$

Since $j \in S_m$, we have $j \in V_{S_m}(P) = S_{m+1}$, which either contradicts the definition of m or determines $m = t - 1$. In this case, it must be that $j \in S_t$ and we obtain a contradiction. This concludes the induction argument and the proof. ■

The previous result allows us to derive a characterization result regarding the convergence of the average OWA.

Corollary 2 *Given a TM $\{\alpha_S\}$, the following two conditions are equivalent:*

1. *The CQM $\{A_S\}$ is convergent.*

2. *For all non-empty sets $U, V \subseteq N$ such that $U \subseteq V$, it holds*

$$|U| \, \alpha_U \geq (|V| - 1) \, \alpha_V.$$

Proof: We initially show that (1) implies (2). Suppose, by way of contradiction, that there exist two non-empty sets $U, V \subseteq N$ such that $U \subset V$ and $|U|\,\alpha_U < (|V|-1)\,\alpha_V$. Consider the profile P defined by:

$$p_{ij} = \begin{cases} 0, & \text{if } j \in N \setminus V \text{ or } \Big(i \in N \setminus U \text{ and } j \in V \setminus U\Big), \\ \alpha_U, & \text{if } i \in U \text{ and } j \in V \setminus U, \\ 1, & \text{if } j \in U. \end{cases}$$

It is straightforward to see that

$$a_N(P,j) = \begin{cases} 1, & \text{if } j \in U, \\ \dfrac{|U|\,\alpha_U}{n-1}, & \text{if } j \in V \setminus U, \\ 0, & \text{if } j \in N \setminus V. \end{cases}$$

Taking into account that $j \in A_N(P) \Leftrightarrow a_N(P,j) \geq \alpha_N$, we have

$$S_2 = A_N(P) = \begin{cases} U, & \text{if } \alpha_N > \dfrac{|U|\,\alpha_U}{n-1}, \\ V, & \text{if } \alpha_N \leq \dfrac{|U|\,\alpha_U}{n-1}. \end{cases}$$

On the other hand,

$$a_U(P,j) = \begin{cases} 1, & \text{if } j \in U, \\ \alpha_U, & \text{if } j \in V \setminus U, \\ 0, & \text{if } j \in N \setminus V \end{cases}$$

and

$$a_V(P,j) = \begin{cases} 1, & \text{if } j \in U, \\ \dfrac{|U|\,\alpha_U}{|V|-1}, & \text{if } j \in V \setminus U, \\ 0, & \text{if } j \in N \setminus V. \end{cases}$$

By $|U|\,\alpha_U < (|V|-1)\,\alpha_V$, we have $a_V(P,j) < \alpha_V$ for every $j \in V \setminus U$ and, therefore, $A_V(P) = U$.

Taking into account the two possible cases:

- $\alpha_N > \dfrac{|U|\,\alpha_U}{n-1}$: $S_1 = N,\ S_2 = U,\ S_3 = V,\ S_4 = U,\ S_5 = V,\ \ldots$
- $\alpha_N \leq \dfrac{|U|\,\alpha_U}{n-1}$: $S_1 = N,\ S_2 = V,\ S_3 = U,\ S_4 = V,\ S_5 = U,\ \ldots$

we can conclude that the committees' sequence does not converge, which is a contradiction.

To prove that (2) implies (1), we only need to show that the weights associated with the average OWA satisfy the condition (1) established in Proposition 2. But clearly, for all positive integers m, u, v such that $m \leq u \leq v$, we have

$$\sum_{i=1}^{m} w_i^u = \frac{m}{u} = \frac{m}{v}\frac{v}{u} = \sum_{i=1}^{m} w_i^v \frac{v}{u}$$

as desired. ■

We complete these results with a discussion of the maximum operator. This function generalizes the dichotomous case corresponding to an *invitation rule*, where one approval of a member of the committee is sufficient to qualify an individual.

Proposition 3 *Given a TM $\{\alpha_S\}$, the following two conditions are equivalent:*

1. *The CQM $\{B_S\}$ is convergent.*

2. *For all non-empty sets $U, V \subseteq N$ such that $U \subseteq V$ it holds $\alpha_U \geq \alpha_V$.*

Proof: To prove that (1) implies (2), suppose, by way of contradiction, that 2) is not true. In this case, there exist two non-empty sets $U \subset V$ such that $\alpha_U < \alpha_V$. Consider the profile P defined by:

$$p_{ij} = \begin{cases} 0, & \text{if } j \in N \setminus V, \\ \alpha_U, & \text{if } j \in V \setminus U, \\ 1, & \text{if } j \in U. \end{cases}$$

It is easy to see that

$$S_2 = B_N(P) = \begin{cases} V, & \text{if } \alpha_U \geq \alpha_N \\ U, & \text{if } \alpha_U < \alpha_N. \end{cases}$$

Similarly, we can check that $B_U(P) = V$ and $B_V(P) = U$. Therefore, $\{B_S\}$ is not convergent and we arrive to a contradiction.

To prove that (2) implies (1), it is sufficient to note that any committee sequence is decreasing for $\{B_S\}$ and therefore, convergent. ■

5 Illustrative example

According to Corollary 1, the CQMs associated with the *worst*, *median* and *olympic* CEMs are not convergent. This means that there exist profiles such that the associated committees' sequences are not convergent. However, there can exist profiles where the associated committees' sequences are convergent.

Let us suppose there is a group $N = \{1, \ldots, 10\}$ of members and the following profile of individual opinions:

$$P = \begin{pmatrix} 1 & 1 & .7 & .6 & .3 & .5 & .6 & .2 & .2 & .4 \\ 1 & 1 & .8 & .5 & .2 & .6 & .7 & .4 & .1 & .7 \\ .9 & .8 & .8 & .7 & .4 & .6 & .6 & .6 & .6 & .7 \\ 1 & 1 & .9 & .8 & .5 & .6 & .6 & .6 & .4 & .7 \\ .8 & .7 & .9 & .6 & .6 & .5 & .3 & .7 & .3 & .8 \\ .7 & .8 & .8 & .7 & .4 & .9 & .4 & .5 & .4 & .7 \\ 1 & 1 & .8 & .7 & .2 & .5 & .9 & .4 & .6 & .8 \\ .5 & .5 & .6 & .4 & .3 & .4 & .3 & .7 & .2 & .2 \\ .6 & .4 & .6 & .7 & .4 & .5 & .6 & .3 & .9 & .4 \\ 1 & 1 & 1 & .5 & .3 & .7 & .8 & .2 & .3 & .7 \end{pmatrix}.$$

Table 4 shows the outcomes provided by the *best*, *worst*, *median*, *average* and *olympic* CEMs for $S = N$.

Table 4: Rounded values provided by $v_N(P, j)$ for the *best*, *worst*, *median*, *average* and *olympic* CEMs

j	1	2	3	4	5	6	7	8	9	10
$b_N(P, j)$	1	1	1	.7	.5	.7	.8	.7	.6	.8
$w_N(P, j)$	.5	.4	.6	.4	.2	.4	.3	.2	.1	.2
$m_N(P, j)$	.9	.8	.8	.6	.3	.5	.6	.4	.3	.7
$a_N(P, j)$	.83	.8	.78	.6	.33	.54	.54	.43	.34	.6
$o_N(P, j)$	.86	.83	.79	.61	.33	.54	.54	.43	.34	.63

Recall that, given a CQM $\{V_S\}$, the committees' sequence $\{S_t\}$ is defined by $S_1 = N$ and $S_{t+1} = V_{S_t}(P)$. For the sake of the exposition, we do not explicit all the values that should be calculated to express committees' sequences.

In Tables 5 and 6 we include the limit set, if there exists, for the CQMs associated with the *best*, *worst*, *median*, *average* and *olympic* CEMs given several possible values of constant thresholds, $\alpha_S = \alpha$ for every $S \subseteq N$. We indicate the stage where the limit is reached by means of the corresponding set S_t.

It is important to note that only in one of the 30 cases we have analyzed, the corresponding sequence does not converge: the *worst* CQM for $\alpha = .6$: $S_1 = N$, $S_{2m} = \{3\}$ and $S_{2m+1} = N \setminus \{5\}$ for every positive integer m. Moreover, in 22 cases the limit is reached in the stage S_2, in 4 it is reached in the stage S_3, and in 3 cases the limit is reached in stage S_4.

6 Concluding remarks

In the recursive decision procedure introduced in this paper, individuals show their opinions on the members of the society by means of assessments within the unit interval. For each

Table 5: Limit sets for *best*, *worst* and *median* CQMs

α	Best	Worst	Median
.3	$S_2 = N$	$S_3 = N \setminus \{5, 8, 9\}$	$S_2 = N$
.4	$S_2 = N$	$S_2 = N \setminus \{5, 8, 9\}$	$S_2 = N \setminus \{5, 9\}$
.5	$S_2 = N$	$S_4 = \{1, 2, 3, 4, 6\}$	$S_2 = N \setminus \{5, 8, 9\}$
.6	$S_2 = N \setminus \{5\}$	It does not exist	$S_3 = N \setminus \{5, 8, 9\}$
.7	$S_3 = N \setminus \{5, 8, 9\}$	$S_2 = \emptyset$	$S_2 = \{1, 2, 3, 10\}$
.8	$S_2 = \{1, 2, 3, 7, 10\}$	$S_2 = \emptyset$	$S_2 = \{1, 2, 3\}$

Table 6: Limit sets for *average* and *olympic* CQMs

α	Average	Olympic
.3	$S_2 = N$	$S_2 = N$
.4	$S_2 = N \setminus \{5, 9\}$	$S_2 = N \setminus \{5, 9\}$
.5	$S_2 = N \setminus \{5, 8, 9\}$	$S_2 = N \setminus \{5, 8, 9\}$
.6	$S_4 = N \setminus \{5, 6, 8, 9\}$	$S_4 = N \setminus \{5, 6, 8, 9\}$
.7	$S_2 = \{1, 2, 3\}$	$S_3 = \{1, 2, 3, 10\}$
.8	$S_2 = \{1, 2\}$	$S_2 = \{1, 2\}$

committees' evaluation mechanism and each family of thresholds, a committees' qualification mechanism is defined that generates a subset of qualified individuals in each stage. It is worth emphasizing that individuals only show their opinions once, and the recursive procedure sequentially generates a new group of qualified individuals taking into account only the opinions of the qualified individuals in the previous stage.

We have mainly considered some specific EOWAs and we have obtained results about the convergence of the associated committees' qualification mechanisms. So, we have found necessary conditions for ensuring the convergence of EOWA mechanisms, being non convergent those EOWAs based on the worst, the median and the olympic procedures. We have also obtained some sufficient conditions ensuring the convergence of EOWA mechanisms. Through them, some necessary and sufficient conditions for the convergence of the EOWAs based on the average and the best opinions have been obtained.

The sequential evaluation could be also be debated and tested for other families of operators. For their connection with the present study, we do not want to avoid to mention Linguistic OWAs (LOWAs) operators, presented in Herrera and Herrera-Viedma [7] or Herrera, Herrera-Viedma and Verdegay [8]. Notice how the use of these different operators would require dimmensional extensions of them, as in the case of EOWAs.

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