Social Choice and Welfare manuscript No.
(will be inserted by the editor)

Characterizing best-worst voting systems in the scoring context

José Luis García-Lapresta[1], **A.A.J. Marley**[2], **Miguel Martínez-Panero**[3⋆]

[1] [3]PRESAD Research Group, Dep. of Applied Economics, University of Valladolid, Avda. Valle de Esgueva 6, 47011 Valladolid, Spain
(e-mail: {lapresta,panero}@eco.uva.es)

[2] Dep. of Psychology, University of Victoria, PO Box 3050 STN CSC, Victoria BC V8W 3P5, Canada; Centre for the Study of Choice, University of Technology, Sydney PO Box 123 Broadway, NSW 2007 Australia (e-mail: ajmarley@uvic.ca)

Received: date / Revised version: date

Abstract An increasing body of theoretical and empirical work on discrete choice considers a choice design in which a person is asked to select both the best and the worst alternative in an available set of alternatives, in contrast to more traditional tasks such as where the person is asked to: select the best alternative; select the worst alternative; rank the alternatives. Here we consider voting systems motivated by such "best-worst" choice; characterize a class of "best-worst" voting systems in terms of a set of axioms in the context of scoring rules; and discuss briefly possible extensions to approval-disapproval systems.

Keywords and Phrases: Scoring rules, Approval voting, Disapproval voting, Best-worst voting, Plurality, Antiplurality.

JEL Classification Numbers: D71, D72.

Send offprint requests to: José Luis García-Lapresta.

⋆ The authors are grateful to Miguel A. Ballester for his suggestions. García-Lapresta and Martínez-Panero gratefully acknowledge the funding support of the Spanish Ministerio de Educación y Ciencia (Project SEJ2006-04267/ECON), ERDF and Junta de Castilla y León (Consejería de Educación y Cultura, Project VA092A08). Marley's research has been supported by Australian Research Council Discovery Grant DP034343632 to the University of Technology, Sydney, for Louviere, Street and Marley, and Natural Science and Engineering Research Council Discovery Grant 8124 to the University of Victoria for Marley.

1 Introduction

Usually, voters find it difficult to linearly order a reasonable number of feasible alternatives. For example, Dummett [8, p. 243] says,

> "If there are, say, twenty possible outcomes, the task of deciding the precise order of preference in which he ranks them may induce a kind of psychological paralysis in the voter; and, for the tellers, the labour of reckoning the preference scores becomes very tedious. We have, therefore, to devise new or modified procedures for use in this case"[1].

However, there is reasonable evidence (see below) that voters can easily (provided they are not indifferent between all the alternatives) select reliably the best and the worst alternative, and, perhaps, rank the p–best and q–worst alternatives for "small" p and q.

Finn and Louviere [9] proposed, and studied, a discrete choice task in which a person selects both the best and the worst alternative in an available (sub)set of alternatives. Since the publication of that paper, interest in, and use of, such best-worst choice tasks has been increasing, with two empirical applications receiving "best paper" awards (Cohen [6]; Cohen and Neira [7]). Despite the increasing use of the approach, Marley and Louviere [15] develop the first detailed theoretical properties of probabilistic models of best-worst choice; Marley [14] summarizes those, and other, theoretical and empirical work. In fact, Marley and Louviere [15] also present some basic results on the "optimality" of scoring rules in the estimation of parameters in probabilistic models of best-worst choice; that work can be interpreted, in the voting context, as assuming a restricted domain for the possible voting profiles. In this paper, we approach the problem deterministically and without domain restrictions.

To find axiomatic characterizations of voting systems is a relevant issue in Social Choice Theory. In fact, as pointed out by Merlin [17], Arrow's theorem [1] can be understood as an axiomatization of dictatorship. Given that Arrow's theorem is usually considered a negative result, May's theorem [16] (which characterizes simple majority) is commonly considered as the first in a long list of characterization theorems of voting theory (for references see Merlin [17] and Marchant [13], among others). In this tradition, we focus our attention on characterizing a class of best-worst voting systems in the context of scoring rules; the latter are of major importance in the voting literature (see Chebotarev and Shamis [4] for a referenced survey).

The remainder of the paper is organized as follows. Section 2 introduces standard and extended scoring rules and their specialization to best-worst voting; Section 3 characterizes general and basic best-worst voting; and Section 4 discusses open problems associated with generalizations of best-worst voting.

[1] Of course, the concern regarding "the labor of reckoning the preference scores" no longer carries much weight.

2 Scoring Rules: General and for Best-Worst Voting

In what follows we assume *situations*[2] in which voters' preferences over a set of n feasible alternatives $X = \{x_1, \ldots, x_n\}$ are given by linear orders. In this context, a *scoring rule* is defined by a vector of scores $(s_1, \ldots, s_n) \in \mathbb{R}^n$, with $s_1 \geq \cdots \geq s_n$ and $s_1 > s_n$, where for each voter's ranking, s_1 points are assigned to the top-ranked alternative, s_2 points to the second-ranked alternative, and so on. The alternative(s) with the largest total score, across voters, is (are) the winner(s)[3].

Given a scoring rule with score vector $(s_1, \ldots, s_n)$, and $a, b \in \mathbb{R}$ such that $a > 0$, the scoring rule with vector of scores $(s'_1, \ldots, s'_n)$, where $s'_i = as_i + b$ for all $i = 1, \ldots, n$, is equivalent to the previous one, in the sense that they provide the same social outcomes.

A scoring rule $(s_1, \ldots, s_n)$ is *standard*[4] if $s_n \geq 0$, otherwise, it is *extended*. Clearly, every extended scoring rule $(s_1, \ldots, s_n)$ is equivalent to the standard one $(s'_1, \ldots, s'_n)$, where $s'_i = s_i - s_n$. Even more, every scoring rule $(s_1, \ldots, s_n)$ (standard or extended) is equivalent to a standard one $(s'_1, \ldots, s'_n)$ where $s'_1 = 1$ and $s'_n = 0$ – simply take $s'_i = (s_i - s_n)/(s_1 - s_n)$.

As discussed in Section 1, usually a person finds it difficult to reliably rank order a reasonable number n of feasible alternatives, but there is considerable evidence from discrete choice experiments that (provided the person is not indifferent between all the alternatives) he/she can select reliably the best and the worst alternative. Also, to the extent that voters respond reliably in elections that require the selection of the best alternative (plurality) [respectively, the worst alternative (antiplurality)], one might expect that they can vote reliably in elections that use best-worst voting. We therefore now discuss scoring rules for (ranked) 1–best and 1–worst voting, followed in Section 3 by a characterization of the corresponding *social choice function* (a social choice function assigns, to each situation, a nonempty subset of X).

Let α, δ be a pair of positive scores[5]; we explain, below, why we constrain the scores to be positive. The *(ranked)* 1*–best* 1*–worst voting procedure* $[\alpha\,;\,\delta]$ is the voting system where, for each voter, just α (respectively, δ) points are assigned to that voter's first-ranked (respectively, last-ranked) alternative. The alternative(s) with the largest score difference – i.e., the difference between the total α score and the total δ score for the alternative across voters – is (are) the winner(s). As the voting rule is intended to depend on both the best and the worst votes, each of α and δ must be

[2] According to Gärdenfors [11], a situation is a set consisting of one preference order for each voter.

[3] For all the voting procedures and scoring rules that we consider, ties can occur. If this causes difficulty to reach the final social decision, then some additional procedure (e.g., random) is required.

[4] This definition differs from that of Woeginger [21] who allows sign free scores.

[5] We use α (resp., δ) as a reminder that the scores are for (ranked) **a**pproval (resp., **d**isapproval) votes.

positive (rather than just nonnegative). In particular, the excluded $[\alpha; 0]$ (respectively, $[0; \delta]$) corresponds to plurality (respectively, antiplurality).

Clearly, 1–best 1–worst, defined by $[\alpha; \delta]$, corresponds to the extended scoring rule $(\alpha, 0, \ldots, 0, -\delta)$, with an equivalent standard scoring rule $(1, \frac{\delta}{\alpha+\delta}, \ldots, \frac{\delta}{\alpha+\delta}, 0)$. We call this *general* 1−*best* 1−*worst*, i.e., the only restriction is that $\alpha > 0, \delta > 0$. Clearly, the relative size of α to δ corresponds to the weight (importance) that the scoring procedure puts on the best versus the worst votes. The special case $[\alpha; \alpha]$, $\alpha \neq 1$, coincides with $[1; 1]$ because their respective extended scoring versions $(\alpha, 0, \ldots, 0, -\alpha)$ and $(1, \ldots, -1)$ are both equivalent to the same standard scoring rule $(1, \frac{1}{2}, \ldots, \frac{1}{2}, 0)$; we call this case *basic* 1–*best* 1–*worst* and add a compact characterization of it after characterizing the general case. Taking into account again their standard scoring versions, presented above, it is easy to see that two 1–best 1–worst voting systems $[\alpha; \delta]$ and $[\alpha'; \delta']$ are equivalent if and only if $\alpha'\delta = \alpha\delta'$. In this way, if $\alpha \neq \delta$, then $[\alpha; \delta]$ is essentially different to $[1; 1]$.

It is important to note that the above use of an extended scoring rule over all the alternatives in the formulation of the (ranked) 1−best 1−worst voting procedure does not entail that voters rank order (all) the alternatives. An exactly parallel situation arises, for instance, with plurality, where voters only indicate their best alternative (they do not need to provide a ranking), but scoring rules over all the alternatives are used in its axiomatic characterization (see Richelson [19]).

3 Characterization of 1–Best 1–Worst Voting

We now develop two characterizations of 1–best 1–worst voting: one for the general case $[\alpha; \delta]$, the other for the basic case $[1\,; 1]$. Our approach follows that used in the fundamental paper of Young [23], where he proved that a scoring rule is a social choice function characterized by the following axioms[6]:

A *Anonymity*: There is an egalitarian consideration for the agents.
N *Neutrality*: There is a symmetric status for each alternative.
R *Reinforcement*: If two disjoint subsets of voters have at least one common alternative among their winners, then all and only such common alternatives keep on being winners for the joined set of voters[7].

[6] For a formal account of conditions generally considered "desirable" for social choice functions (these axioms among them), see Young [22, 23]. It is worth mentioning that another characterization of scoring rules was given by Smith [20].

[7] According to Young [22], this property is a kind of Pareto condition on subsets of voters. It is also considered by Fishburn [10] in his characterization of approval voting, and by Young and Levenglick [24] in their characterization of Kemeny's rule. For other names for this condition (*consistency*, *separability*, etc.), see Merlin [17, p. 95].

C *Continuity*: If two disjoint sets of voters U and V select x and y as winners, respectively, then x is a winner for the superset $(mU) \cup V$ for m sufficiently large[8].

There is a very large literature characterizing voting systems in the scoring context. For example: the Borda rule has been characterized by Young [23] and Merlin [17, Theorem 7]; Richelson [19], Lepelley [12], Ching [5] and Merlin and Naeve [18] each characterize plurality rule (in different ways); and Merlin [17] and Baharad and Nitzan [2] characterize anti-plurality (in different ways).

Given the successes of this approach, our goal is to develop properties of best-worst voting that allow us to say that its associated scoring rule is determined by the fulfillment of these conditions in addition of those proposed by Young. We succeed in this goal for general $1-$best $1-$worst voting $[\alpha; \delta]$ and for the basic case $[1; 1]$; the first requires three conditions, the second a single condition, in addition to Young's.

First we present the three properties that, together with Young's, are necessary and sufficient to characterize general 1–best 1–worst voting $[\alpha; \delta]$.

TSM *Top Strict Monotonicity*: If x is a non-unique winner in a situation where at least one voter considers x to be the best alternative, then x would not be a winner in the situation obtained where just this voter changes his opinion only about x (preserving his pairwise preferences about the other alternatives).

BSM *Bottom Strict Monotonicity*: If x is a non-unique winner in a situation where at least one voter does not consider x to be the worst alternative, then x would not be a winner in the situation obtained where just this voter changes his opinion about x, and decides that x is the worst alternative in the second situation (preserving his pairwise preferences about the other alternatives).

IMA *Independence of Middle Alternatives*: The winner(s) in a situation are preserved if one or more voters change their opinions about alternatives other than those they have selected as their personal best and worst (i.e., if pairwise preferences containing the best or the worst alternative do not change in a new situation, then the winner(s) are the same).

The above three properties are related to those used in characterizations of other voting systems. For instance, that of plurality (respectively, antiplurality) uses bottom invariance (respectively, top invariance) plus the standard (Young) conditions (see Barberà and Dutta [3] and Merlin [17]). However, in Merlin [17] "top" means above the winner, whereas our "top" (i.e., "best") means a winner (the same applies, in a symmetric manner, for "bottom"). In fact, our top and bottom conditions are somewhat related to May's [16] *positive responsiveness*, and our **IMA** can be understood as an invariance condition in the sense of Merlin [17].

[8] In this axiom, for U a set of voters and their votes, (mU) means m copies of those voters and votes. This *Archimedian property* was previously considered by Smith [20].

Theorem 1. An extended scoring rule satisfies **TSM**, **BSM** and **IMA** if and only if it is a general 1–best 1–worst voting system $[\alpha; \delta]$.

PROOF. Obviously, every 1–best 1–worst voting system satisfies the required properties.

For sufficiency, we consider the cases $n = 2, 3$ in detail so that the reader will more easily understand the general case $n \geq 4$.

Notice that if $n = 2$, just the definition of scoring rule entails $s_1 > s_2$, given that $\alpha > 0, \delta > 0$, and it is routine to check that **TSM**, **BSM** (and **IMA** trivially) hold.

If $n = 3$, **IMA** trivially holds, and it will be proven that **TSM** and **BSM** entail $s_1 > s_2 > s_3$.

Consider the situation:

Voter 1	Voter 2	Voter 3
x_1	x_2	x_3
x_2	x_3	x_1
x_3	x_1	x_2

It is clear that all the alternatives obtain the same total score: $s_1+s_2+s_3$, so all of them are winners.

Now we modify the previous situation in two cases:

1. The first voter interchanges the first and the second alternatives, and the opinions of the other voters do not change:

Voter 1	Voter 2	Voter 3
x_2	x_2	x_3
x_1	x_3	x_1
x_3	x_1	x_2

   According to **TSM**, now x_1 is not a winner. Thus, the total score of x_1, $2s_2 + s_3$, is smaller than the total score of at least other alternative. Taking into account that the total score of x_2 is $2s_1 + s_3$, and the total score of x_3 is $s_1 + s_2 + s_3$, then we have that either $2s_2 + s_3 < 2s_1 + s_3$ or $2s_2 + s_3 < s_1 + s_2 + s_3$. In either case, $s_1 > s_2$.

2. The third voter interchanges the second and the third alternatives, and the opinions of the other voters do not change:

Voter 1	Voter 2	Voter 3
x_1	x_2	x_3
x_2	x_3	x_2
x_3	x_1	x_1

   According to **BSM**, now x_1 is not a winner. Thus, the total score of x_1, $s_1 + 2s_3$, is smaller than the total score of at least other alternative. Taking into account that the total score of x_2 is $s_1 + 2s_2$, and the total score of x_3 is $s_1 + s_2 + s_3$, then we have that either $s_1 + 2s_3 < s_1 + 2s_2$ or $s_1 + 2s_3 < s_1 + s_2 + s_3$. In either case $s_2 > s_3$.

Note that the obtained extended scoring rule, (s_1, s_2, s_3), where $s_1 > s_2 > s_3$, is equivalent to $(s_1 - s_2, 0, s_3 - s_2)$ which defines the 1–best 1–worst voting system $[\alpha\,;\delta]$, where $\alpha = s_1 - s_2$ and $\delta = s_2 - s_3$.

Finally, suppose that $n \geq 4$ and consider an extended scoring rule with associated vector of scores $(s_1, \dots, s_n)$ such that $s_1 \geq \cdots \geq s_n$ and $s_1 > s_n$.

In order to prove $s_1 > s_2 = \cdots = s_{n-1} > s_n$, consider the following situation where, for $k = 1, .., n-1$, the elements in row $k+1$ are obtained from row k by moving the element in column 1 in row k to column n in row $k+1$, and moving the element in column j, $j \neq 1$, in row k to column $j-1$ in row $k+1$.

Voter 1	Voter 2	$\cdots$	Voter $n-1$	Voter n
x_1	x_2	$\cdots$	x_{n-1}	x_n
x_2	x_3	$\cdots$	x_n	x_1
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
x_k	x_{k+1}	$\cdots$	x_{k-2}	x_{k-1}
x_{k+1}	x_{k+2}	$\cdots$	x_{k-1}	x_k
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
x_{n-1}	x_n	$\cdots$	x_{n-3}	x_{n-2}
x_n	x_1	$\cdots$	x_{n-2}	x_{n-1}

It is clear that all the alternatives obtain the same total score: $s_1 + \cdots + s_n$, so all of them are winners[9].

Now we modify the previous situation in the following cases:

1. The first voter interchanges the first and the second alternatives, and the opinions of the other voters do not change:

Voter 1	Voter 2	$\cdots$	Voter $n-1$	Voter n
x_2	x_2	$\cdots$	x_{n-1}	x_n
x_1	x_3	$\cdots$	x_n	x_1
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
x_{n-1}	x_n	$\cdots$	x_{n-3}	x_{n-2}
x_n	x_1	$\cdots$	x_{n-2}	x_{n-1}

According to **TSM**, now x_1 is not a winner. Thus, the total score of x_1, $2s_2 + s_3 + \cdots + s_n$, is smaller than the total score of at least one other alternative. Taking into account that the total score of x_2 is $2s_1 + s_3 + \cdots + s_n$, and that of each of $x_3, \dots, x_n$ is $s_1 + \cdots + s_n$, then we have that either $2s_2 + s_3 + \cdots + s_n < 2s_1 + s_3 + \cdots + s_n$ or $2s_2 + s_3 + \cdots + s_n < s_1 + \cdots + s_n$. In either case, $s_1 > s_2$.

2. The second voter interchanges x_1 and x_n, and the opinions of the other voters do not change:

[9] Note that it is also true for any social choice function satisfying anonymity and neutrality.

Voter 1	Voter 2	$\cdots$	Voter $n-1$	Voter n
x_1	x_2	$\cdots$	x_{n-1}	x_n
x_2	x_3	$\cdots$	x_n	x_1
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
x_{n-1}	x_1	$\cdots$	x_{n-3}	x_{n-2}
x_n	x_n	$\cdots$	x_{n-2}	x_{n-1}

According to **BSM**, now x_n is not a winner. Thus, the total score of x_n, which is $s_1 + \cdots + s_{n-2} + 2s_n$, is smaller than the total score of at least one other alternative. Taking into account that the total score of x_1 is $s_1 + \cdots + s_{n-2} + 2s_{n-1}$, and that of each of $x_2, \ldots, x_{n-1}$ is $s_1 + \cdots + s_n$, then either $s_1 + \cdots + s_{n-2} + 2s_n < s_1 + \cdots + s_{n-2} + 2s_{n-1}$ or $s_1 + \cdots + s_{n-2} + 2s_n < s_1 + \cdots + s_n$. In either case $s_{n-1} > s_n$.

3. The first voter interchanges x_k and x_{k+1}, successively for $k = 2, \ldots, n-2$, and the opinions of the other voters do not change:

Voter 1	Voter 2	$\cdots$	Voter $n-1$	Voter n
x_1	x_2	$\cdots$	x_{n-1}	x_n
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
x_{k+1}	x_{k+1}	$\cdots$	x_{k-2}	x_{k-1}
x_k	x_{k+2}	$\cdots$	x_{k-1}	x_k
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
x_n	x_1	$\cdots$	x_{n-2}	x_{n-1}

According to **IMA**, all the alternatives remain winners. In particular, x_k and x_{k+1} should have the same score. Then,

$$(s_1 + \cdots + s_n) - s_k + s_{k+1} = (s_1 + \cdots + s_n) + s_k - s_{k+1}.$$

Consequently, $s_k = s_{k+1}$ for $k = 2, \ldots, n-2$, i.e., $s_2 = s_3 = \cdots = s_{n-1}$.

Thus, we have $s_1 > s_2 = s_3 = \cdots = s_{n-1} > s_n$. This extended[10] scoring rule, $(s_1, s_2, \ldots, s_n)$, is equivalent to the standard $(s_1 - s_2, 0, \ldots, 0, s_n - s_2)$ which defines the 1–best 1–worst voting system $[\alpha\,;\delta]$, where $\alpha = s_1 - s_2$ and $\delta = s_2 - s_n$. ■

Proposition 1. TSM, **BSM** and **IMA** are independent.
Proof.

1. Plurality satisfies **TSM** and **IMA**, but not **BSM**.
2. Antiplurality satisfies **BSM** and **IMA**, but not **TSM**.
3. The Borda rule satisfies **TSM** and **BSM**, but not **IMA**. ■

Now we present a characterization of the basic 1–best 1–worst voting system $[1\,;1]$ by means of just one property, in addition to Young's. This additional condition is related to Young's [22] cancellation condition.

TBC *Top Bottom Cancellation*: In any situation where each alternative considered the best by one voter is cancelled by the same alternative considered the worst by another voter, all the alternatives win.

[10] Notice that no sign conditions on the scores are obtained from the imposed properties.

Theorem 2. An extended scoring rule satisfies **TBC** if and only if it is the basic 1–best 1–worst voting system $[1\,;1]$.
PROOF. Obviously, $[1\,;1]$ satisfies **TBC**.

Notice that if $n = 2$, just the definition of scoring rule entails $s_1 > s_2$, and it is routine to check that holds.

Now, if $n \geq 3$, consider the following situation:

Voter 1	Voter 2
x_1	x_n
x_2	x_2
$\cdots$	$\cdots$
x_{n-1}	x_{n-1}
x_n	x_1

The total score of each of x_1 and x_n is $s_1 + s_n$, and that of x_i, $i = 2, \ldots, n-1$, is $2s_i$. By **TBC** all the alternatives win, and so all of them have the same total score: $s_1 + s_n = 2s_2 = \cdots = 2s_{n-1}$.

This extended scoring rule, $(s_1, \frac{s_1+s_n}{2}, \ldots, \frac{s_1+s_n}{2}, s_n)$, is equivalent to $(s_1 - s_n, \frac{s_1-s_n}{2}, \ldots, \frac{s_1-s_n}{2}, 0)$, and to $(2, 1, \ldots, 1, 0)$, and to $(1, 0, \ldots, 0, -1)$, which is just $[1\,;1]$. ∎

4 Discussion

In this paper, we have assumed that voters can easily select both their "best" and their "worst" alternative in a consistent fashion. However, one might ask whether this is the case. This question is partially answered, in a positive manner, by the success of the best-worst method in discrete choice experiments (see Section 1). Also, to the extent that voters can respond reliably in elections that require a voter to select the best alternative (plurality) [respectively, the worst alternative (antiplurality)], one might expect that they can vote reliably in elections that use 1−best 1−worst voting. Nonetheless, if each voter may have a weak order, rather than a linear order, over the alternatives, then it may be preferable not to ask a voter to select the best, worst, or best and worst candidate(s), as several candidates may be tied – either as best or as worst – in the voter's weak order. In such a case, approval-disapproval voting[11] seems appropriate, and an interesting open problem (as far as we know) is to characterize that voting method.

Finally, if it is reasonable to assume that a voter can partially order the set of n alternatives, beyond the "best" and the "worst", then it is of interest to explore the characterization of the more general *ranked p−approval*

[11] *Approval-disapproval voting* allows each voter to approve of any number of alternatives k $(0 \leq k \leq n)$ and to disapprove of any number of alternatives l $(0 \leq l \leq n)$, where k and l are selected by the voter, with the constraints that the set of approved alternatives is disjoint from the set of disapproved alternatives, and $1 \leq k + l \leq n$. The natural social choice function assumes that the alternative(s) with the highest difference score – its total approval votes minus its total disapproval votes – is (are) the winner(s).

$q-$disapproval voting system - where each voter approves of a common number, p, of alternatives and disapproves of a common number, q, of alternatives, with similar constraints on the sets of approved and disapproved alternatives to those concerning approval-disapproval voting.

References

1. Arrow, K.J.: Social Choice and Individual Values. 2nd edition [1st edition: 1951], Wiley, New York (1963)
2. Baharad, E., Nitzan, S.: The inverse plurality rule – an axiomatization. Social Choice and Welfare **25**, 173–178 (2005)
3. Barberà, S., Dutta, B.: Implementability via protective equilibria. Journal of Mathematical Economics **10**, 49–65 (1982)
4. Chebotarev, P.Y., Shamis, E.: Characterizations of scoring methods for preference aggregation. Annals of Operations Research **80**, 299–332 (1998)
5. Ching, S.: A simple characterization of plurality rule. Journal of Economic Theory **71**, 298–302 (1996)
6. Cohen, S.: Maximum difference scaling: Improved measures of importance and preference for segmentation. Sawtooth Software Conference Proceedings, Sawtooth Software, Inc. 530 W. Fir St. Sequim, WA (www.sawtoothsoftware.com), pp. 61–74 (2003)
7. Cohen, S., Neira, L.: Measuring preference for product benefits across countries: Overcoming scale usage bias with maximum difference scaling. Paper presented at the Latin American Conference of the European Society for Opinion and Marketing Research, Punta del Este, Uruguay (2003). Reprinted in Excellence in International Research: ESOMAR, Amsterdam, The Netherlands, pp. 1–22 (2004)
8. Dummett, M.: Voting Procedures. Clarendon Press, Oxford (1984)
9. Finn, A., Louviere, J.J.: Determining the appropriate response to evidence of public concern: The case of food safety. Journal of Public Policy and Marketing **11**, 12–25 (1992)
10. Fishburn, P.C.: Axioms for approval voting: Direct proof. Journal of Economic Theory **19**, 180–85 (1978)
11. Gärdenfors, P.: Positionalist voting functions. Theory and Decision **4**, 1–24 (1973)
12. Lepelley, D.: Une caracterérisation du vote à la majorité simple. RAIRO Operations Research 26, 361–365 (1992)
13. Marchant, T.: An axiomatic characterization of different majority concepts. European Journal of Operational Research **179**, 160–173 (2007)
14. Marley. A. A. J.: The best-worst method for the study of preferences: Theory and Applications. Proceedings of XXIX International Congress of Psychology. Psychology Press, 2009 (in press)
15. Marley, A.A.J., Louviere, J.J.: Some probabilistic models of best, worst, and best-worst choices. Journal of Mathematical Psychology **49**, 464–480 (2005)
16. May, K.O.: A set of independent necessary and sufficient conditions for simple majority decision. Econometrica **20**, 680-684 (1952)
17. Merlin, V.: The axiomatic characterizations of majority voting and scoring rules. Mathématiques et Sciences Humaines. Mathematics and Social Sciences **161**, 87–109 (2003)

18. Merlin, V., Naeve, J.: Implementation of social choice functions via demanding equilibria. Diskussionspapiere aus dem Institut für Volkswirtschaftslehre der Universität Hohenheim from Department of Economics, University of Hohenheim, No 191/2000 (2000)
19. Richelson, J.T.: A characterization result for the plurality rule. Journal of Economic Theory **19**, 548–550 (1978)
20. Smith, J.H.: Aggregation of preferences with variable electorate. Econometrica **41**, 1027–1041 (1973)
21. Woeginger, G.H.: A note on scoring rules that respect majority in choice and elimination. Mathematical Social Sciences **46**, 347–354 (2003)
22. Young, H.P.: An axiomatization of Borda's rule. Journal of Economic Theory **9**, 43–52 (1974)
23. Young, H.P.: Social choice scoring functions. SIAM Journal on Applied Mathematics **28**, 824–838 (1975)
24. Young, H.P., Levenglick, A.: A consistent extension of Condorcet's election principle. SIAM Journal on Applied Mathematics **35**, 285–300 (1978)