Comparison of Three Voting Systems

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Range Voting (RV), RV Median, Approval Voting (AV)

Assume candidates A, B, C, . . . are graded on a 6-point scale (proposed by Balinski and Laraki, 2007). For simplicity, I ignore candidates C, D, E, . . ., whom I assume do worse than A and B under both RV and RV Median, which Balinski and Laraki called "majority judgment" voting.

Thm. *RV and RV Median may elect a candidate preferred by only <u>one</u> voter*. **Proof** (by example):

RV

Voter

Candidate	1	2	3	4	5	Total
A	6	1	1	1	1	<u>10</u>
B	1	2	2	2	2	9

—Candidate A, favored only by voter 1, wins.

RV Median

Voter

Candidate	1	2	3	4	5	Median
A	6	6	3	2	2	3
B	5	5	4	1	1	<u>4</u>

—Candidate B, favored only by voter 3, wins.

Plausible Scenarios under Approval Voting (AV) (examples from previous slide)

RV

Voter

Candidate	1	2	3	4	5	Total
A	**6**	1	1	1	1	<u>10</u>
B	1	**2**	**2**	**2**	**2**	9

—Whereas candidate A won under RV, candidate B wins under AV 4 to 1 (boldface), given voters always vote for a most-preferred candidate and never vote for a least-preferred candidate (proved to be optimal in Brams and Fishburn, 1978, 1983).

RV Median

Voter

Candidate	1	2	3	4	5	Median
A	**6**	**6**	3	**2**	**2**	3
B	5	5	**4**	1	1	<u>4</u>

—Whereas candidate B won under RV Median, candidate A wins under AV 4 to 1 (boldface), given voters always vote for a most-preferred candidate and never vote for a least-preferred candidate

Conclusions

1. It is disturbing that a candidate preferred by only one voter can win under both RV and RV Median.

2. Insofar as elections boil down to 2-candidate contests—as many do—this cannot happen under AV, because a candidate preferred by more voters will be approved of by at least as many, at least if the voters are *sincere* (i.e., don't skip over preferred candidates).

3. Because AV leaves it up to the voter to draw the line between acceptable and unacceptable candidates, it is works well when voters have different grading standards:

	Voter						
Candidate	1	2	3	4	5	Total	Median
A	**6**	**6**	2	2	2	<u>18</u>	2
B	4	4	**3**	**3**	**3**	17	<u>3</u>

As the underscores show, RV selects candidate A, whereas RV Median selects candidate B. B would also be selected if this were viewed as a 2-candidate contest, in which each voter would vote only for his or her preferred candidate (boldface).

4. In a multicandidate contest, whether one or more candidates are acceptable should depend on the *voters'* judgments of acceptability—not the aggregation method—especially if they have different standards of grading.

Manipulability

It is well-known that AV and RV are manipulable. But so is RV Median if a voter's truthful scores for two candidates are both high or both low.

For example, suppose the median scores for Gore and Bush in 2000 were 4 and 3. Then a voter who ranked one candidate high (say, 5 or 6) and one low (say, 1 or 2) could not change the outcome by being insincere.

But a voter who ranked them both high or both low would have an incentive to bottom-rank his *less-preferred* candidate and top-rank his *more-preferred* candidate, which is equivalent to AV.

A Realistic Example with 100 Voters?

49 Gore Voters: Gore (6) > Nader (3)* > Bush (1)*
3 Nader Voters: Nader (6) > Gore (2)* > Bush (1)*
48 Bush Voters: Bush (6) > Gore (2)* > Nader (1)

Under RV, the average scores are Gore (3.52), Bush (3.40), Nader (2.13), so Gore wins. Under RV Median, the medians are Gore (2), Bush (1), and Nader (3)—all starred—so Nader wins. Under AV, the biggest gaps are below 6, so it seems everybody would approve only of his or her first choice, and Gore would squeak by.

But Nader's supporters, knowing that he is out of the running, would have good reason to vote strategically for Gore as well. Similarly, under RV Median, these voters would presumably rate Gore more highly so he would be their safety choice (which they can anticipate needing!).

No-Show Paradox (Fishburn and Brams, 1983)

No-show paradox in reverse: It occurs when new voters (or their ballots) show up—indicating a favorite candidate who won without them—but the inclusion of their ballots causes that candidate to lose! The Hare system of single transferable vote (STV), called instant runoff voting (IRV) in the United States, is vulnerable, but so is RV Median (RV and AV are not).

Example

Five voters give the following scores to three candidates, and candidate A wins with the highest median score (4), which (medians underscored):

Candidate A	Scores: 6 4 **4** 2 1
Candidate B	Scores: 6 5 **3** 2 1
Candidate C	Scores: 6 3 **2** 1 1

Two new voters show up, and both score candidates A, B, and C, respectively, as follows: 6 5 1.

Candidate A	Scores: 6 **4** 4 2 1	6 6
Candidate B	Scores: 6 **5** 3 2 1	5 5
Candidate C	Scores: 6 3 2 **1** 1	1 1

But now candidate B has the highest median score (5), even though the two new voters most favored candidate A.