A compromise between Majority Judgement and Range Voting

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Workshop on New Approaches to Voting and Social Choice

Tilburg, May 25-26, 2009

Majority Judgement

Michel BALINSKI, Rida LARAKI: MAJORITY JUDGEMENT

- The Majority Judgement (2007)
  http://ceco.polytechnique.fr/jugement-majoritaire.html
- A theory of measuring, electing and ranking
  Proceedings of the National Academy os Sciences of the United States of America 104, pp. 8720-8725 (2007)
- Election by Majority Judgement: Experimental evidence
  Ecole Polytechnique – Centre National de la Recherche Scientifique, Cahier 2007-28 (2007)

A proposal for voting in political elections
by means of linguistic assessments

↓

median + breaking ties

Range/Utilitarian Voting

Warren D. Smith: RANGE VOTING

- Range voting (2000)
  http://www.math.temple.edu/~wds/homepage/rangevote.pdf

Claude Hillinger: UTILITARIAN VOTING

- Voting and the cardinal aggregation of judgments
  SEMECON, University of Munich (2004)
- The case for utilitarian voting
  Department of Economics, University of Munich, Discussion paper 2005-11 (2005)

Two proposals for voting in political elections
by means of numerical scales

↓

arithmetic mean

Criticisms on Majority Judgement

- **W. D. Smith** (2007): On Balinski & Laraki's "Majority Judgement" median-based range-like voting scheme
  http://rangevoting.org/MedianVrange.html
- **D. S. Felsenthal, M. Machover**: The Majority Judgment voting procedure: A critical evaluation
  Forthcoming in *Homo Oeconomicus*
- **J. L García-Lapresta, M. Martínez-Panero** (2008): Sorting alternatives into linguistic classes and their aggregation
  Computational Intelligence in Decision and Control, World Scientific, Singapore, pp. 531-536
- **J. L. García-Lapresta, M. Martínez-Panero** (2009): Linguistic-based voting through centered OWA operators
  Forthcoming in *Fuzzy Optimization and Decision Making*
- **H. Nurmi** (2009): Voting Theory
  Forthcoming

Majority Judgement *versus* Range Voting

Majority Judgement

- It uses the median as aggregation operator
- Breaking ties $\longrightarrow$ a lot of cases

Range Voting

- It does not use linguistic information but numerical values
- It uses the arithmetic mean as aggregation operator
- It does not need to break ties

Ballot used in the Orsay experiment

Bulletin de vote du « jugement majoritaire »

Pour présider la France,
ayant pris tous les éléments en compte,
je juge en conscience que ce candidat serait :

	Très Bien	Bien	Assez Bien	Passable	Insuffisant	A Rejeter
Olivier Besancenot						
Marie-George Buffet						
Gérard Schivardi						
François Bayrou						
José Bové						
Dominique Voynet						
Philippe de Villiers						
Ségolène Royal						
Frédéric Nihous						
Jean-Marie Le Pen						
Arlette Laguiller						
Nicolas Sarkozy						

Cochez une seule mention dans la ligne de chaque candidat.

Ne pas cocher une mention dans la ligne d'un candidat revient à le Rejeter.

The Orsay experiment: Official, MJ and RV rankings

CANDIDATE	OFFICIAL	MJ	RV
Royal	1	2	2
Sarkozy	2	3	3
Bayrou	3	1	1
Le Pen	4	12	11
Besancenot	5	5	5
Villiers	6	10	9
Voynet	7	4	4
Buffet	8	6	6
Bové	9	7	7
Laguiller	10	8	8
Nihous	11	9	10
Schivardi	12	11	12

Notation

- $V = \{1, \ldots, m\}$ set of voters $(m \geq 2)$
- $X = \{x_1, \ldots, x_n\}$ set of alternatives $(n \geq 2)$
- $L = \{l_1, \ldots, l_g\}$ ordered set of linguistic terms $(g \geq 2)$

  $l_1 < \cdots < l_g$

Example

l_1	l_2	l_3	l_4	l_5	l_6
to reject	poor	acceptable	good	very good	excellent

Notation

- A **profile** is a matrix $m \times n$ with coefficients in L

$$\begin{pmatrix} a_1^1 & \cdots & a_j^1 & \cdots & a_n^1 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_1^i & \cdots & a_j^i & \cdots & a_n^i \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_1^m & \cdots & a_j^m & \cdots & a_n^m \end{pmatrix}$$

  where $a_j^i \in L$ is the assessment that voter i assigns to x_j
- $\mathcal{P}$ set of profiles

Majority Judgement

$$\begin{pmatrix} a_1^1 & \cdots & a_j^1 & \cdots & a_n^1 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_1^i & \cdots & a_j^i & \cdots & a_n^i \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_1^m & \cdots & a_j^m & \cdots & a_n^m \end{pmatrix} \longmapsto (l(x_1), \ldots, l(x_j), \ldots, l(x_n))$$

$$l(x_j) = f(a_j^1, \ldots, a_j^m) \qquad j = 1, \ldots, n$$

Middlemost condition (Galton, 1907)

$l(x_j) \in L$ must satisfy

$$\#\{i \in V \mid a_j^i \geq l(x_j)\} \geq \frac{m}{2} \quad \text{and} \quad \#\{i \in V \mid a_j^i \leq l(x_j)\} \geq \frac{m}{2}$$

Majority Judgement $\longrightarrow$ f median

The *middlemost* condition in small electorales

Remark (after Galton, 1907)

If $l(x_j) \in L$ fulfills the *middlemost* condition, this social grade cannot be objectionable by an absolute majority of voters for being either too high or too low

Remark

There always exists $l(x_j) \in L$ for each $x_j \in X$ verifying the *middlemost* condition, but such a grade might not be necessarily unique

Notation

$L(x_j)$ set of terms satisfying the *middlemost* condition

- Balinski – Laraki proposal: $l(x_j) = \min L(x_j)$
  In large electorates usually $|L(x_j)| = 1$

The *middlemost* condition in small electorales

Our adjustment

$$l(x_j) = \begin{cases} \text{median } L(x_j) & \text{if } |L(x_j)| \text{ is odd} \\ \text{median } L(x_j) \setminus \{\max L(x_j)\} & \text{if } |L(x_j)| \text{ is even} \end{cases}$$

Example

TR	P	A	G	VG	E	**MJ**
$4+1$	0	0	0	0	5	TR
4	1	0	0	0	5	P
4	0	1	0	0	5	A
4	0	0	1	0	5	G

A median voter could become a kind of dictator

The *middlemost* condition in small electorales

Example

TR	P	A	G	VG	E	MJ	Adjustment
5	0	0	0	0	5	TR	A
4	1	0	0	0	5	P	G
4	0	1	0	0	5	A	G
4	0	0	1	0	5	G	VG

The collective grade under our adjustment might not have been assessed by any voter

Our proposal

- Majority Judgement is very sensitive towards the median voter
- Range Voting is very sensitive towards extreme assessments

García-Lapresta, J.L., Martínez-Panero, M.
Linguistic-based voting through centered OWA operators
Forthcoming in *Fuzzy Optimization and Decision Making*

1. Voters assign a linguistic term to each candidate
2. Individual assessments are aggregated by means of centered OWA operators and the 2-tuple approach
3. The outcome is a 2-tuple for each candidate: a linguistic term plus a number (for breaking ties)
4. Candidates are sorted and ranked

The 2-tuple approach (Herrera – Martínez, 2000)

$\langle L\rangle = L \times [-0.5, 0.5)$ is the 2-tuple set associated with L

The function $\Delta : [1, g] \longrightarrow \langle L\rangle$ is given by

$$\Delta(\beta) = (l_h, \alpha) \;\text{ with }\; \begin{cases} h = \text{ round } (\beta) \\ \alpha = \beta - h \end{cases}$$

where *round* assigns to β the integer $h \in \{1, \dots, g\}$ closest to β

Example

l_1	l_2	l_3	l_4	l_5	l_6
to reject	poor	acceptable	good	very good	excellent

$$\langle L\rangle \equiv [1, 6]$$

$$\Delta(3.8) = (\text{good}, -0.2) \quad \Delta(4.3) = (\text{good}, 0.3)$$

OWA operators (Yager, 1988)

Let $F_{\mathbf{w}} : \mathbb{R}^m \longrightarrow \mathbb{R}$ be the OWA operator associated with the weighting vector $\mathbf{w} = (w_1, \dots, w_m) \in [0, 1]^m$, such that $\sum_{i=1}^m w_i = 1$

$$F_{\mathbf{w}}(\beta_1, \dots, \beta_m) = w_1 \cdot \beta_{(1)} + \cdots + w_m \cdot \beta_{(m)}$$

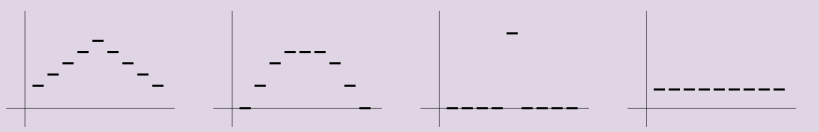
where $\beta_{(i)}$ is the i-th greatest number of $\beta_1, \dots, \beta_m$

Centered OWA operators (after Yager, 2007)

Let $F_{\mathbf{w}}$ be the OWA operator associated with the weighting vector $\mathbf{w} = (w_1, \ldots, w_m) \in [0,1]^m$, such that $\sum_{i=1}^m w_i = 1$

We say that $F_{\mathbf{w}}$ is **centered** if the following two conditions are satisfied:

1. *Symmetry*
$$w_i = w_{m+1-i} \text{ for every } i \in \{1, \ldots, [\tfrac{m}{2}]\}$$
2. *Decaying*
$$w_i \leq w_j \text{ whenever } i < j \leq \left[\tfrac{m+1}{2}\right] \text{ or } i > j \geq \left[\tfrac{m+1}{2}\right]$$



Our proposal

$\pi : L \longrightarrow \{1, \ldots, g\}$ is defined by $\pi(l_h) = h$ for $h = 1, \ldots, g$

Definition

Let $F_{\mathbf{w}}$ the centered OWA operator associated with the weighting vector $\mathbf{w} = (w_1, \ldots, w_m)$

The mapping $G_{\mathbf{w}} : \mathcal{P} \longrightarrow \langle L \rangle^n$ is defined by

$$G_{\mathbf{w}}(P) = (v(x_1), \ldots, v(x_n))$$

where

$$v(x_j) = \Delta(F_{\mathbf{w}}(\pi(a_j^1), \ldots, \pi(a_j^m)))$$

is the collective assessment on x_j

Our proposal

$$L^m \longrightarrow \langle L \rangle$$
$$(a_j^1, \ldots, a_j^m) \mapsto v(x_j)$$

satisfies *anonymity, unanimity, monotonicity, compensativeness* and *self-duality*

The previous definition induces a voting system, called the **$F_{\mathbf{w}}$-procedure**

It generates a weak order on the set of alternatives:

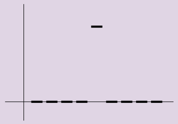
$$x_i \succcurlyeq x_j \Leftrightarrow v(x_i) \succcurlyeq v(x_j)$$

$$(l_k, \alpha_k) \succcurlyeq (l_{k'}, \alpha_{k'}) \Leftrightarrow \begin{cases} k > k' \\ \text{or} \\ k = k' \text{ and } \alpha_k \geq \alpha_{k'} \end{cases}$$

Majority Judgement and Range Voting as $F_{\mathbf{w}}$-procedures

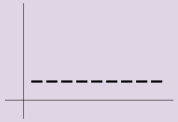
If m is odd, then Majority Judgement is the $F_{\mathbf{w}}$-procedure corresponding to

$$w_i = \begin{cases} 1 & \text{if } i = \frac{m+1}{2} \\ 0 & \text{otherwise} \end{cases}$$



Range Voting is the $F_{\mathbf{w}}$-procedure corresponding to

$$w_1 = \cdots = w_m = \frac{1}{m}$$



Example 1

$V = \{1, \ldots, 9\}, \; X = \{x_1, x_2\}, \; L = \{l_1, \ldots, l_7\}$

	1	2	3	4	5	6	7	8	9
x_1	l_4	l_7	l_7	l_3	l_7	l_3	l_3	l_7	l_3
x_2	l_5	l_6	l_6	l_1	l_6	l_1	l_1	l_6	l_1

- Eight out of nine agents prefer x_1 to x_2 and only one agent prefers x_2 to x_1
- Under Majority Judgement x_2 defeats x_1 because the median of the assessments are l_5 and l_4 respectively

x_1	l_7	l_7	l_7	l_7	l_4	l_3	l_3	l_3	l_3
x_2	l_6	l_6	l_6	l_6	l_5	l_1	l_1	l_1	l_1

- Under a $F_{\mathbf{w}}$-procedure

$$x_1 \succ x_2 \;\Leftrightarrow\; w_5 < 0.6$$

- Under Range Voting x_1 defeats x_2 $(w_5 = 0.11 < 0.6)$

Example 2

$V = \{1, \ldots, 5\}, \; X = \{x_1, x_2\}, \; L = \{l_1, \ldots, l_7\}$

	1	2	3	4	5
x_1	l_7	l_7	l_7	l_7	l_1
x_2	l_6	l_6	l_6	l_6	l_7

- Four out of five agents prefer x_1 to x_2 and only one agent prefers x_2 to x_1
- Under Majority Judgement x_1 defeats x_2 because the median of the assessments are l_7 and l_6, respectively

x_1	l_7	l_7	l_7	l_7	l_1
x_2	l_7	l_6	l_6	l_6	l_6

- Under a $F_{\mathbf{w}}$-procedure

$$x_1 \succ x_2 \;\Leftrightarrow\; w_1 < 0.14$$

- Under Range Voting x_2 defeats x_1 $(w_1 = 0.2 > 0.14)$

Concluding remarks

- Majority Judgement is not suitable for small electorates (committees)
- Majority Judgement needs a breaking ties process that uses more information than just the median
- Range Voting is very sensitive towards extreme opinions (outliers)
- The proposed voting system is very flexible and allows us to adapt it to each specific scenario