

RIGHTS REVISITED, AND LIMITED

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ABSTRACT. One of the most justly famous result of social choice theory is Sen's Theorem on the impossibility of a Paretian liberal. In two recent papers, Salles introduced the notion of limited rights both in an aggregation function framework and in a social choice function framework. He then proved Sen-type impossibility theorems. In the aggregation function framework an individual has a 'right' if whenever she prefers an option (social state), say a , to another social state, say b , the social preference ranks a before b . Salles proposed to consider the following weakening. Rather than a being socially ranked before b , he suggests that b should not be ranked before a . In the social choice framework, a framework which was introduced later on and was thought to be more or less equivalent to the aggregation function framework, if the individual prefers a to b , b must not be chosen from any set to which a belongs. Salles's weakening amounts to say that if it happens that b be chosen, then a must be chosen too. In the present paper, we will describe from an intuitive point of view the technical results obtained by Salles in the light of the distinction between possibility and obligation, and we will present a research program based on the use of tools borrowed from modal logic.

1. INTRODUCTION

In two recent papers, Salles (2008, 2009) introduced a notion of limited rights¹ first in an aggregation function framework (2008), then in the framework of social choice rules. Our purpose in this paper, is to provide a general exposition of these results from an intuitive viewpoint in a way that imitates in some sense the famous non-starred chapters in Sen (1970b).² The studies of rights, freedom, liberalism within social choice theory originated in Sen's magisterial paper (1970a). In this short contribution, Sen demonstrates that there is an incompatibility between some weak form of collective rationality of social preference, a Pareto unanimity condition and some specific form of what Sen called at that time liberalism, which can also possibly be interpreted as an unequal distribution of power or/and a violation of the so-called neutrality condition. Since 1970, rights etc. have been considered within other paradigms. For instance rights have been introduced in game forms by Gärdenfors (1981, 2005), Gaertner, Pattanaik and Suzumura (1992), Peleg (1998), Suzumura (2006) and freedom (liberty) has been mainly analyzed in the context of opportunity sets following the pioneering paper of Pattanaik and Xu (1990). Saari and Pétron (2006) and Li and Saari (2008) have recently revisited the foundational framework of Sen and Gibbard (1974) by examining the informational structure of the aggregation procedures.³

In section 2, we will present the necessary concepts for individuals and for society emphasizing the aggregation function approach compared to the social choice approach. In Section 3 we will describe Sen's theorem in both frameworks and give examples. We will allude to the Cartesian product structure to show how it can help to formalize the notion of personal sphere. Section 4 will deal with limited rights as introduced by Salles in his

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¹The first to have presented this notion in an unfortunately unpublished paper is Edi Karni (1974).

²In Sen's book the starred chapters are the formal chapters and non-starred chapters are informal.

³See also Saari (2008).

recent papers. The two final sections will be devoted to a general discussion about welfareism and possible extensions of the analysis of rights within social choice using modal logic.

2. NECESSARY CONCEPTS

We consider a group of people who have preferences over options that will be identified with social states. To simplify, we may consider a finite group of individuals who rank (with possible ties) a finite set of social states according to their preferences. Each individual having a ranking, the social choice question is to find either a *social* preference or some way to choose from any part of the set of social states. Procedures to obtain a social preference from a list of individual preferences will be called *aggregation functions* and procedures to obtain a choice for any set of options will be called *social choice functions*.⁴

The social preference can be a ranking or some other type of relations with less demanding rationality properties. For instance, with a ranking if someone prefers a to b and is indifferent between b and c she prefers a to c , and if she is indifferent between a and b and between b and c , she is indifferent between a and c (this property is the transitivity of indifference–ties). We can consider also, for social preferences, the case when only strict preferences are transitive (when a is preferred to b and b to c , a is preferred to c), the indifference relation being no more transitive (this has been called quasi-transitivity by Sen). One can also imagine that indifference can be in some sense *partly* transitive as in the case of semi-orders or interval orders. The strict social preference can be acyclic which means that there is no cycles of the type a preferred to b , b preferred to c , c preferred to d and d preferred to a .⁵ The acyclicity condition is strongly related in formal developments in choice theory to the existence of a choice within any finite subset of the set of options.

The other framework is to have, given the rankings of individuals, a rule to choose from any set of social states with the obligation that some social state must be chosen in **this** set. In decision theory and standard microeconomics, the choice of elements from a set can be defined by reference to a binary relation meaning ‘at least as good as’ in which case the chosen elements are the best according to this relation (provided that they exist). But the chosen elements be can given and then one can infer a binary relation on the basis of choice. This is the central topic of revealed preference theory (the choice made by some individual reveals her preference).

3. SEN’S THEOREM

In the social preference framework, Sen considers the case where the collective rationality is rather weak, that is strict social preferences are supposed to be acyclic (this means, in particular, that indifference is not supposed to be transitive and even that strict preference is not supposed to be transitive—it is possible that a be preferred to b and b to c with an indifference between a and c). The so-called **Pareto condition** states that a is socially preferred to b whenever all individuals prefer a to b . In general people think that the Pareto condition is not questionable. It has however an important consequence. The aggregation functions (or, for that matter, the social choice functions) cannot be constant functions provided there is a sufficient diversity in the individual preferences (it is sufficient that there exist two options a and b for which individuals can possibly prefer a to b or b to a , since in such cases it is possible that all individuals prefer a to b or all individuals prefer b to a and, in the aggregation functions setting, a is socially preferred to b or b is socially preferred to a). If the aggregation function is a constant function,

⁴We use the word ‘function’ even if the word ‘correspondence’ is more common. Here our function is accordingly a set-valued function.

⁵Here the cycle involves four options, but acyclicity excludes all cycles whatever the number of options.

the outcome is a fixed social preference whatever the individual preferences. This would be the case when the fixed preference has been determined by a moral or religious code. The **liberalism condition** attributes to each individual i some specific power over two social states, say a_i and b_i , viz. a_i is socially preferred to b_i whenever individual i prefers a_i to b_i and b_i is socially preferred to a_i whenever i prefers b_i to a_i . Sen's result can be obtained for a weaker form of liberalism, called **minimal liberalism**, where there are only two individuals who are endowed with the just described specific power.⁶ Sen's Theorem shows that there is an incompatibility between the acyclicity of the social preference, the Pareto condition and the condition of minimal liberalism provided that there is a sufficient diversity of the individual rankings.

In the choice-theoretic framework, the Pareto condition takes the following form. In a subset of social states to which a and b belong, b is not chosen whenever all individuals prefer a to b , and the condition of minimal liberalism regarding, say, individual i and social states a and b , amounts to say that b is not chosen in a set to which a belongs whenever individual i prefers a to b , and that a is not chosen in a set to which b belongs whenever individual i prefers b to a . It can then be shown that there is an incompatibility between the existence of a choice procedure, the Pareto condition and minimal liberalism provided that there is a sufficient diversity of the individual rankings.

Let us consider a famous example introduced by Sen about the reading of *Lady Chatterley's Lover*. This example involves two individuals, let us call them Mr. Prude and Mr. Lascivious and three options, social states, a , b and c , where a is a social state where Mr. Prude reads *Lady Chatterley's Lover*, b is the same social state except that Mr. Lascivious reads the book and c , again, is the same social state except that no one reads the book. One can find reasons for Mr. Prude ranking the options with c being top-ranked, a being ranked in second position and b in the third. It seems obvious that option c which amounts to censorship has his preference, but ranking a before b means that he does not want that Mr. Lascivious can benefit in any way from reading the book and he is ready to sacrifice himself in this matter. Mr. Lascivious will rank the three options in the following way. One can imagine that he is against censorship so that he will rank c at the bottom of his ranking. Being not only lascivious but also slightly sadistic, he has pleasure to imagine Mr. Prude having to read the book, and, accordingly, he top-ranks option a . Now, the liberalism condition can be applied as far as Mr. Prude is concerned to options a and c . This implies that in the social preference c will be ranked before a . This same condition regarding Mr. Lascivious will be applied to options b and c . This implies that in the social preference b will be ranked before c . But both individuals have ranked a before b , and as a consequence of the Pareto condition, a will be ranked before b in the social preference. One can see then that a is ranked before b which is ranked before c which is ranked before a , which is a cycle.

We believe that there is some ambiguity in this example, because it is rather unclear whether when, say, Mr. Prude reads the book, this will definitely prevent Mr. Lascivious from reading it. One way to deal with this difficulty is to introduce a Cartesian product structure for the options so that each option is in fact an ordered pair. In this framework option a where Mr. Prude reads the book is either Mr. Prude reads the book **and** Mr. Lascivious does not read the book or both Mr. Prude **and** Mr. Lascivious read the book. If for an ordered pair (x, y) , the first coordinate, x , refers to Mr. Prude and the second, y , to Mr. Lascivious, one can write (r, n) for Mr. Prude reads the book and Mr. Lascivious does not read it, (r, r) for both read the book, (n, r) for Mr. Prude does

⁶We will only consider this minimal version of liberalism, even in its weak form, in the next section, since all impossibilities obtained for minimal liberalism have implicit corollaries for liberalism. Accordingly, we will not use systematically the term 'minimal', 'liberalism' meaning also, from now on, 'minimal liberalism'.

not read the book and Mr. Lascivious reads the book, and (n, n) for no one reads the book. We now have four rather than three options and it seems that Sen's a is (r, n) , b is (n, r) and c is (n, n) . Suppose now that Mr. Prude ranks the four options as, from the most preferred to the least preferred, $(n, n), (r, n), (n, r), (r, r)$ and that Mr. Lascivious as $(r, r), (r, n), (n, r), (n, n)$. With this structure it seems natural that Mr. Prude has the 'power' conferred by the liberalism condition over options (r, n) and (n, n) since in both cases Mr. Lascivious's situation is the same (he does not read the book), but also over options (n, r) and (r, r) since in both cases Mr. Lascivious reads the book. Similarly Mr. Lascivious should have the 'power' over (n, n) and (n, r) since in both cases Mr. Prude does not read the book and also over (r, n) and (r, r) since in both cases Mr. Prude reads the book. Given this and the Pareto condition ((r, n) is ranked before (n, r) by both individuals), one obtains two cycles rather than one cycle, viz. a cycle $(n, n), (r, n), (n, r), (n, n)$ and a cycle $(r, r), (r, n), (n, r), (r, r)$.⁷

In the choice-theoretic framework, in Sen's original example, since a is preferred to b by both individuals, the Pareto condition requests that b be not chosen in the set made of a , b and c . But a is not chosen either, since given the power of Mr. Prude over a and c and the fact that Mr. Prude ranks c before a , a cannot be chosen in a set to which c belongs. Similarly, with Mr. Lascivious and options b and c , c cannot be chosen in a set to which b belongs. In conclusion, there is no option that can be chosen among the three options a , b and c .

In the modified Sen's example, if we consider a set of options made of $(n, n), (r, n), (n, r)$, since (r, n) is ranked before (n, r) by both individuals, the Pareto condition excludes (n, r) as a possible choice. Since Mr. Prude prefers (n, n) to (r, n) , (r, n) is excluded and since Mr. Lascivious prefers (n, r) to (n, n) , (n, n) is also excluded so that there is no choice in this three-option set. Similarly, if we consider a set of options made of $(r, r), (r, n), (n, r)$, the Pareto condition excludes (n, r) , and the liberalism condition as applied to Mr. Lascivious and options (r, r) and (r, n) excludes option (r, n) and applied to Mr. Prude and options (n, r) and (r, r) excludes option (r, r) so that we are left with nothing as a possible choice in this set.

It should be noted that with the Cartesian product structure and liberalism being defined as a power conferred to individuals when other individuals' situation is unmodified it is possible to construct examples where cycles are obtained without using the Pareto condition (see, for instance, Salles (2000)). Furthermore, this Cartesian product structure seems to be an interesting formalization of the idea of *personal sphere* often associated to Mill's major work (1859).

In his book (1970b), Sen suggests other examples. For instance, he proposes options involving sleeping positions (on the back or on the belly) or involving the color of kitchen walls. He also develops his analysis and give further examples in a number of papers which are collected in his book of 2002.⁸ Incidentally, we will use sleeping positions examples in the sequel.

4. LIMITED RIGHTS

Salles's purpose in his two papers (2008, 2009) was formally to study a weakening of the conditions associated with the notion of individual liberty. We both consider that this condition even if restricted to two individuals was rather strong in the mathematical

⁷This variation on Sen's example was presented in Salles (1996). It is also in Hausman and McPherson (2006). The first edition of Hausman and McPherson appeared in 1996.

⁸See also Sen (1976, 1983).

framework that was developed and only its interpretation, in our view, not only make it acceptable but even obvious. In his comments to a paper of Brunel (now Pétron) and Salles (1998), Hammond (1998) writes:

In the social choice rule approach ..., local dictatorship become a desideratum, provided that the 'localities' are appropriate. Our feelings of revulsion should be reserved for non-local dictatorships, or local dictatorships affecting issues that should not be treated as personal.

Of course, we share this opinion, but there is nothing in the basic mathematical framework that guarantees this personal aspect (in contrast with a suitable Cartesian product structure). In this basic framework it is however possible to weaken dictatorships. What Salles demonstrates in his two papers is that this weakening does not offer a very interesting escape route from Sen's negative results, this being particularly true in the framework of social choice rules as compared with the aggregation function framework.

In this section, we will first present as simply as possible the technical results and then we will provide examples. Let us explain first why we think that it is important to consider a weakening of the liberalism conditions. There is some ambiguity in the social choice treatment of liberalism regarding, in the choice-theoretic framework, either the *ability* to throw out an option or the *obligation* to throw out an option, or, in the aggregation function framework regarding the fact that the individual endowed with a right over two options either necessarily imposes his strict preference over these two options to society or not. It seems that the liberalism as stated is about *obligation* or *necessity*.⁹ We thought that some remedy to what could be considered as a slight defect could be to impose that the social preference be not the reverse of the individual preference of the individual endowed with the mentioned right or, in the choice framework that the less preferred option by the individual either be thrown out or if not thrown out that the preferred option be also chosen.

4.1. The social preference framework. The Pareto condition remains identical: whenever all individuals prefer option x to option y , then x is ranked before y in the social preference. Let us now explain how the liberalism condition, as applied to only two individuals is weakened. Let us suppose that the weak liberalism condition pertains to individual i regarding options a and b . Whenever i prefers one option, say, a , to the other option, b , then b cannot be ranked before a in the social preference. Given that social preference is supposed to be complete, this is equivalent to saying that either a is ranked before b or there is a tie at the social preference level between a and b (in such a situation we will say that a is socially *at least as good as* b). To have the weak liberalism condition, one, of course, needs to have something similar for another individual, say, j , and options c and d . Now, if we consider a rule based on unanimity, often called a Pareto extension rule, which says that option x is socially preferred to option y if all individuals prefer x to y and y is socially at least as good as x otherwise, we can see that each individual is endowed with the kind of power attributed by the weak liberalism condition not only over two options but over all options. It is very easy to see that the social strict preference (*preferred to* or *better than*) is transitive so that the kind of possible cycles obtained in Sen's analysis are not obtainable any more. However, if we ask for a stronger form of collective rationality, we get again an impossibility. In Salles (2008), three forms of stronger rationality conditions were considered.

The most standard rationality condition is the transitivity of the social preference, viz. of the relation 'at least as good as' which implies as already mentioned that both the strict preference, 'better than,' and indifference are transitive. The aggregation rule is then a *social welfare function* in the terminology of Arrow (1950, 1951, 1963). What Salles (2008)

⁹See, for clarifying comments, Pattanaik (1996).

demonstrates is that there is no social welfare function satisfying the Pareto condition and the weak liberalism condition provided that the two options over which the two individuals have some power are not identical (this means, of course, that there are at least three options). To see how this is possible, we reconsider Sen's example about *Lady Chatterley's Lover*. Mr. Prude ranks the options in the order c, a, b (c , first, a second and b third) and Mr. Lascivious in the order a, b, c . Because of the weak liberalism condition as applied to Mr. Prude, c is socially as good as a , and as applied to Mr. Lascivious b is socially at least as good as c . Then by transitivity of the relation 'at least as good as,' b is socially as good as a . But the Pareto condition tells us exactly the opposite: b is socially better than a . When a Cartesian structure is introduced in this example we obtain two violations of transitivity (1) since (n, r) is socially at least as good as (n, n) by weak liberalism applied to Mr. Lascivious, (n, n) is socially at least as good as (r, n) by weak liberalism applied to Mr. Prude, and then (n, r) is socially at least as good as (r, n) by transitivity, which is contradicted by (r, n) socially better than (n, r) by the Pareto condition, and (2) since (n, r) is socially at least as good as (r, r) by weak liberalism applied to Mr. Prude, (r, r) is socially at least as good as (r, n) by weak liberalism applied to Mr. Lascivious, and then (n, r) is socially at least as good as (r, n) by transitivity, which is contradicted by (r, n) socially better than (n, r) by the Pareto condition.

Between transitivity of the relation 'at least as good as' and transitivity of the relation 'better than' (where the relation of indifference is not supposed to be transitive), two kinds of relations have been introduced: semi-orders and interval orders (Luce (1959), (2000), Suppes & al. (1989), Fishburn (1985)). Semi-orders and interval orders consider the possibility of, loosely speaking, partial transitivity of indifference. We will give an example for interval orders which is the relation that has the weakest form of rationality for which we obtain an impossibility. The kind of transitivity assumption for interval orders states that the relation 'better than' is (implicitly) transitive and that if we have for four elements a, b, c and d , a better than b , an indifference between b and c , and c better than d , then a will be better than d .¹⁰ We will assume that a is a social state in which individual i sleeps on the back, b is the same social state as a , except that individual i sleeps on the right side, and that c is a social state in which individual j takes a bath in any given morning and d is the same social state as c except that individual j takes a shower in the same given morning. The weak liberalism condition attributes power to individual i as far as social states a and b are concerned, and to individual j as far as social states c and d are concerned. Let us assume that individual i ranks the four social states in the order d, b, a, c and individual j in the order a, c, d, b . Since both individuals rank a before c , and d before b by the Pareto condition a is socially better than c , and d is socially better than b . Since j has a power over c and d and since she prefers c to d , by weak liberalism c is socially as good as d . If (1) this 'socially at least as good as' is in fact 'socially better than,' then by transitivity of 'better than' a is socially better than b . Since individual i prefers b to a and since weak liberalism give her power over a and b , we should have b socially at least as good as a , which contradicts a socially better than b . If now (2) this 'socially at least as good as' is 'there is an indifference between', then since a is socially better than c , there is a social indifference between c and d and b is socially better than d , we must have by the property of interval orders, a socially preferred to b , and we are exactly in the same situation as in the case (1) where b was socially as good as a , that is we obtain a contradiction from the assumption of weak liberalism attributed to individual i regarding social states a and b .

It is shown by Salles (2008) that the results obtained for social welfare functions, semi-order valued functions and interval-order valued functions are very similar, the differences pertaining to the set of social states. For social welfare functions, the two social states over which individuals i and j have power must not be identical. For semi-order valued

¹⁰Given completeness of the relation 'at least as good as,' this implication is equivalent to another implication which is generally used in measurement theory.

functions, we must add that there are at least four social states. For interval-order valued functions, the social states over which individual i and j have power must be distinct (so that there are at least four social states). As shown by the extended version of Sen's example regarding *Lady Chatterley's Lover* having four distinct social states is not a strong requirement. With two elements a and b , one can form four elements in the two-person Cartesian product structure: $(a, b), (b, a), (a, a), (b, b)$.

4.2. The choice-theoretic framework. In the choice-theoretic framework, the Pareto condition remains the same as what we described in Section 3. An option x which is Pareto-dominated by y will not be chosen in a set of options to which y belongs. On the other hand, the weak liberalism entails an important modification. As previously, two individuals i and j have a specific power over two options, a and b for i , and c and d for j . Let us consider individual i . Weak liberalism says that if individual i prefers one of the two options, say, a , to the other, b , then if it happens that b be chosen in a set to which a belongs then a has to be chosen too. For individual j , it says, of course that if j prefers one of the two options, say, c , to the other, d , then if it happens that d be chosen in a set to which c belongs c has to be chosen too.

Let us consider an example that was previously described. The options a and b are identical social states except that in a individual i sleeps on the back and in b she sleeps on the right side. Options c and d are also identical social states except that in c individual j take a bath in any given morning and in d she takes a shower in the same given morning. Let us assume that individual i ranks the social states in the order d, a, b, c and individual j in the order b, c, d, a . To find a rationale behind these rankings, let us imagine the following story. Individual j is individual i 's servant. Individual i knows that if individual j takes a bath in the morning she will be sleepy during all the day and, then, less efficient. Accordingly, she ranks d at the top of her preference and c at the bottom. Since she prefers to sleep on the back she ranks a before b . Individual j knows that sleeping on the back is not that recommended for a good rest and that when individual i is a little tired she is in a bad mood. Consequently, she ranks b at the top of her preference and a at the bottom. Since she prefers taking a bath rather than having a shower, she ranks c before d . This rationale shows very clearly the negative externality phenomenon. It also shows that it is not so easy to justify that people who are, in principle, unconcerned should be indifferent (here, for instance, individual i should be indifferent between c and d). It shows, in fact, the limits of the notion of personal sphere. If we consider a choice over the set composed of the four social states a, b, c and d , since both individuals prefer d to a and b to c , neither a nor c can be chosen by the Pareto condition. We are left with b and d . But since individual i prefers a to b , if b happens to be chosen then a has to be chosen too by weak liberalism applied to individual i . The fact is that a cannot be chosen so b cannot be chosen too. We are left with d . But since individual j prefers c to d , by weak liberalism if d happens to be chosen c has to be chosen too. We know that c being chosen is impossible. In conclusion there is no choice at all. It is shown in Salles (2009) that we still have an impossibility when one social state a or b is either c or d (then a, b, c and d are not distinct) provided that there are at least four social states. When there are only three social states (in this case, of course, one of a or b is either c or d), we can still obtain an impossibility provided that the so-called *Weak Axiom of Revealed Preference* (WARP) is satisfied. WARP was introduced in the form presented here by Arrow (1959, 1984) and studied among others by Schwartz (1976), Sen (1971) and Suzumura (1976). We can explain WARP in a very elementary way. Let us suppose that you are making your weekly shopping in a hypermarket. At the exit, in your shopping trolley there are goods from many different sections including the household products section. Now if we set all the goods aside and ask you to start again your shopping but with the possibility to uniquely visit the household products section, what you will choose will be exactly what

you previously chose in the household products section. This is WARP!

What is remarkable with this result is that, except that we cannot deal with the case of only two social states and that for three-option case we need WARP, it is exactly the result obtained in Section 3. There is no need to consider rationality properties of the WARP type. Now, is there a justification for using this new-weak-form of liberalism? Although we have been criticized because choice can only be uniquely defined, on the basis that it is impossible to have two different social states being chosen, it seems to us that this is rather irrelevant since this is in complete contrast with the standard theory of choice as developed by economists, psychologists, philosophers, mathematicians etc. Furthermore, let us consider our previous example. Let us suppose that individual i slightly prefers a to b , that is she slightly prefers to sleep on the back. In the liberalism version of Section 3, b is then excluded if a can be chosen. If sleeping on the back is a possible option, sleeping on the right side is forbidden for i .¹¹ Since her preference is only slight, this means that she does not dislike sleeping on the right side and from time to time even take pleasure in sleeping also on her right side.¹² With weak liberalism, either the social state including ‘sleeping on the right side’ is excluded or, if not excluded, then the social state including ‘sleeping on the back’ is permissible. We can then truly see the condition of liberalism as a condition giving rights: if individual i has the right to sleep on the right side, she must also have the right to sleep on her back. One can develop the same kind of comments with the bath and shower variant. Individual j may prefer taking a bath to having a shower, but this preference may be very slight. It seems strange in this case that she has no right to have a shower. Weak liberalism says on the other hand that if she has the right to have a shower, she must also have the right to take a bath.

5. DISCUSSION AND REMARKS

Salles (2008) draws a parallel between Arrovian theorems and Sen-type theorems.¹³ From a technical point of view there are major differences. Arrovian theorems are obtained for finite societies (the set of individuals has to be finite) and this is not necessary for Sen-type theorems. Also, there is no need of the controversial condition of independence of irrelevant alternatives for Sen-type theorems. The main difference which is at the origin of the numerous researches in non-welfaristic issues in normative economics is that Sen-type theorems are non-welfaristic. The word *welfarism* is associated with the idea that the goodness of social states are evaluated only on the basis of individual utilities attached to these social states. This leads to the following observation. If we have four social states w , x , y and z and if each individual attributes the same utility to w and to x , and the same utility to y and to z , then the social ranking of w and y must be the same as the social ranking of x and z . This can be generalized to various properties which have been called neutrality properties for aggregation functions defined on lists of individual utility functions and can be extended to aggregation functions defined on lists of individual preferences in which case one obtains intra or inter lists neutrality.¹⁴ For instance, if the preference restriction of each individual i to x and y is related to her preference restriction to z and w , related in the following sense, individual i prefers x to y if and only if she prefers z to w , she prefers y to x if and only if she prefers w to z and she is indifferent between x and y if and only if she is indifferent between z and w , then x is socially preferred to y if and only if z is preferred to w , y is socially preferred to x if and only if w is socially preferred to z and there is a social indifference between x and y if and only if there is a

¹¹If during her sleep, individual i turns to the right side while she was sleeping on the back, one must have a device to take her back to her back!

¹²This also calls our attention to problems related to indeterminacy and vagueness (Piggins and Salles (2007)).

¹³For Arrovian theorems see Blau (1979), Blair and Pollack (1979), Blair & al. (1976) and Suzumura (1983).

¹⁴A remarkable introduction to the non-welfaristic literature is Pattanaik (1994).

social indifference between z and w . The rôles of x and z are similar, as are the rôles of y and w .¹⁵ Intuitively, neutrality means that the names of social states do not matter. To give a simple voting example, if voters are asked to rank the candidates and the voting procedure gives an outcome that is also a ranking, then if candidate a is ranked before candidate b in the outcome ranking, modifying all ballot papers by replacing the name a by the name c and the name b by the name d , given neutrality (which is, in general, satisfied by voting rules), will give an outcome where c will be ranked before d . The liberalism conditions obviously violate neutrality since specific social states are attached to specific individuals. Likewise, anonymity, a condition that says that the name of individuals do not matter, is also violated since some specific individuals have some specific power.

In the choice-theoretic framework, we obtained exactly the same kind of result as Sen's theorem (with the exception of the assumptions on the number of options and on the two options over which the individuals have some power, but we believe that these assumptions are not very constraining).

6. CONCLUSION

In this paper, we have described, in a rather intuitive way, the results obtained by Salles (2008, 2009). We have considered two frameworks. A framework where social preferences were obtained from individual preferences (an aggregation function setting) and a framework where, rather than a social preference, a choice was done over all subsets of possible social states (a social choice function setting—or social choice correspondence setting as it is called by people who do not like to use the word 'function' when the values taken by the 'function' are subsets of a given set). The intuitive presentation has been based on many examples, some of which are quite famous, examples that were totally absent in Salles's papers. We think that the choice-theoretic framework is particularly interesting in the sense that it offers a rather direct generalization of Sen's theorem (which is not the case in the aggregation function setting). The next step in our research will be to study rights within the social choice paradigm by using the concepts of possibility and necessity as introduced in modal logic.¹⁶

REFERENCES

- Arrow, K.J., 1950. A difficulty in the concept of social welfare. *Journal of Political Economy*, 58, p.328-346.
- Arrow, K.J., 1951, 1963. *Social choice and individual values*. New York: Wiley.
- Arrow, K.J., 1959. Rational choice functions and orderings. *Economica*, 26, p.121-127.
- Arrow, K.J., 1984. *Individual choice under certainty and uncertainty. Collected papers of Kenneth J. Arrow, Volume 3*. Oxford: Blackwell.
- Barberà, S., Bossert, W. & Pattanaik, P.K., 2004. Ranking sets of objects. In S. Barberà, P.J. Hammond & C. Seidl, eds. *Handbook of utility theory. Volume 2*. Dordrecht: Kluwer.
- Blair, D.H., Bordes, G., Kelly, J. S. & Suzumura, K., 1976. Impossibility theorems without collective rationality. *Journal of Economic Theory*, 13, p.361-379.
- Blair, D.H. & Pollack, R.A., 1979. Collective rationality and dictatorship: the scope of the Arrow theorem, *Journal of Economic Theory*, 21, p.186-194.
- Blau, J.H., 1979. Semiorders and collective choice. *Journal of Economics Theory*, 21, p.195-203.

¹⁵Formally z is obtained from x and w is obtained from y by a permutation over the finite set of options.

¹⁶See Priest (2008).

- Brunel, A. & Salles, M., 1998. Interpretative, semantic and formal difficulties of the social choice approach to rights. In J.-F. Laslier, M. Fleurbaey & A. Trannoy, eds. *Freedom in economics: new perspectives in normative economics*. London: Routledge.
- Fishburn, P.C., 1985. *Interval orders and interval graphs*. New York: Wiley.
- Gaertner, W., Pattanaik, P.K. & Suzumura, K., 1992. Individual rights revisited. *Economica*, 59, p.161-177.
- Gärdenfors, P., 1981. Rights, games and social choice. *Nous*, 15, p.341-356.
- Gärdenfors, P., 2005. *The dynamics of thought*. Heidelberg: Springer.
- Gibbard, A., 1974. A Pareto consistent libertarian claim. *Journal of Economic Theory*, 7, p.388-410.
- Hammond, P.J., 1998. Some comments on Brunel and Salles. In J.-F. Laslier, M. Fleurbaey & A. Trannoy, eds. *Freedom in economics: new perspectives in normative economics*. London: Routledge.
- Hausman, D.M. & McPherson M.S., 2006. *Economic analysis, moral philosophy, and public policy*, second edition, Cambridge: Cambridge University Press.
- Karni, E., 1974. Individual liberty, the Pareto principle and the possibility of social decision function. Working Paper, The Foerder Institute for Economic Research, Tel-Aviv University.
- Li, L., Saari, D.G., 2008. Sen's theorem: geometric proof, new interpretations. *Social Choice and Welfare*, 31, p.393-413.
- Luce, R.D., 1959. *Individual choice behavior. A theoretical analysis*. New York: Wiley.
- Luce, R.D., 2000. *Utility of gains and losses. Measurement-theoretical and experimental approaches*. Mahwah (New Jersey): Lawrence Erlbaum Associates.
- Mill, J.S., 1859. *On liberty*. London.
- Pattanaik, P.K., 1994. Some non-welfaristic issues in welfare economics. In B. Dutta, ed. *Welfare economics*. Delhi: Oxford University Press.
- Pattanaik, P.K., 1996. On modelling individual rights: some conceptual issues. In K.J. Arrow, A.K. Sen & K. Suzumura, eds. *Social choice reexamined, Volume 2*. London: MacMillan.
- Pattanaik, P.K. & Xu, Y., 1990. On ranking opportunity sets in terms of freedom of choice. *Recherches Economiques de Louvain*, 56, p.383-390.
- Peleg, B., 1998. Effectivity functions, game forms, games, and rights. *Social Choice and Welfare*, 15, p.67-80, and in J.-F. Laslier, M. Fleurbaey, N. Gravel & A. Trannoy, eds. *Freedom in economics: new Perspectives in normative economics*. London: Routledge.
- Piggins, A. & Salles, M., 2007. Instances of indeterminacy. *Analyse & Kritik*, 29, p.311-328.
- Priest G., 2008. *An introduction to non-classical logic: from if to is*, second edition. Cambridge: Cambridge University Press.
- Saari, D.G. & Pétron, A., 2006. Negative externalities and Sen's liberalism theorem. *Economic Theory*, 28, p.265-281.
- Saari, D.G., 2008. *Disposing dictators, demystifying voting paradoxes: social choice analysis*. Cambridge: Cambridge University Press.
- Salles, M., 1996. Discussion of Pattanaik's paper. In K.J. Arrow, A.K. Sen & K. Suzumura, eds. *Social choice reexamined, Volume 2*. London: MacMillan.
- Salles, M., 2000. Amartya Sen. Droits et choix social. *Revue Economique*, 51, p.445-457.
- Salles, M., 2008. Limited rights as partial veto and Sen's impossibility theorem. In P.K. Pattanaik & al., eds. *Rational choice and social welfare, theory and applications: essays in honor of*

Kotaro Suzumura. Heidelberg: Springer.

Salles, M., 2009. Limited rights and social choice rules. In K. Basu & R. Kanbur, eds. *Arguments for a better world, essays in honor of Amartya Sen, Volume I, ethics, welfare, and measurement*. Oxford: Oxford University Press.

Schwartz, T., 1976. Choice functions, "rationality" conditions, and variations on the weak axiom of revealed preference. *Journal of Economic Theory*, 13, p.414-427.

Sen, A.K., 1970a. The impossibility of a Paretian liberal. *Journal of Political Economy*, 78, p.152-157.

Sen, A. K., 1970b. *Collective choice and social welfare*. San Francisco: Holden-Day.

Sen, A. K., 1971. Choice functions and revealed preference. *Review of Economic Studies*, 38, p.307-317.

Sen, A.K., 1976. Liberty, unanimity and rights. *Economica*, 49, p.217-245.

Sen, A.K., 1982. *Choice, welfare and measurement*. Oxford: Blackwell.

Sen, A. K., 2002. *Rationality and freedom*. Cambridge (Mass.): Harvard University Press.

Suppes, P., Krantz, D.H., Luce, R.D. & Tversky, A., 1989. *Foundations of measurement. Volume 2*. New York: Academic Press.

Suzumura, K., 1976. Rational choice and revealed preference. *Review of Economic Studies*, 43, p.149-158.

Suzumura, K., 1983. *Rational choice, collective decisions and social welfare*. Cambridge: Cambridge University Press.

Suzumura, K., 2006. Rights, opportunities, and social choice procedures. In K.J. Arrow, A.K. Sen & K. Suzumura, eds. *Handbook of social choice and welfare, Volume 2*.

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