The Logic of Campaigning

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Language and Political Power

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Umberto Eco

Towards a Semiological Guerrilla Warfare, 1967

Plan of talk

- **Learning via Communication**
- **Applications to the Logic of Campaigning**

In the Coffee Shop

Three people A, B, C walk into a coffee shop. One of them orders cappuccino, one orders tea, and one orders icecream. The waiter goes away and after ten minutes *another* waiter arrives with three cups. “Who has the cappuccino?” “I do,” says A. “Who has the tea?” “I do,” says C.

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Will the waiter ask a third question?”

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When C says that he has the tea, 1 is eliminated.

Now 2 alone is left and the waiter knows that B has the icecream.

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The butler’s remark eliminated S1 and saved her from embarrassment.

Learning from Communication

Observation: Suppose a group of people are commonly aware of a number of possibilities (states) among which they are uncertain. They commonly know some fact ψ if ψ is true of all these possibilities. Now, if a public announcement of some true formula φ is made, then the new situation is obtained by deleting all states *s* where φ is false.

Numerical Foreheads

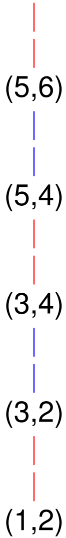
Two players Ann and Bob are told that the following will happen. Some positive integer n will be chosen and *one* of n, $n + 1$ will be written on Ann's forehead, the other on Bob's. Each will be able to see the other's forehead, but not his/her own.

Note that each can see the other's number, but not their own. Thus if Ann has 5 and Bob has 6, then Ann knows that her number is either 5 or 7 and Bob knows that his number is either 6 or 4.

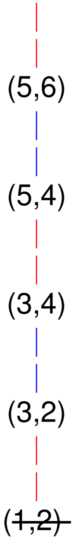
After this is done, they are asked repeatedly, beginning with Ann, if they know what their own number is.

We now look at what is happening when Ann has 5 and Bob has 6.

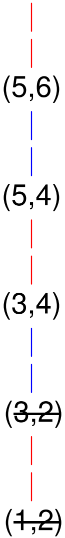
Start situation



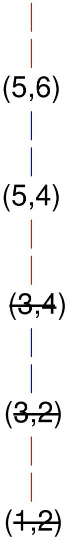
Bob has just said, I don't know my number



Ann said no also



Bob said a second “no”



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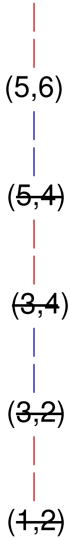
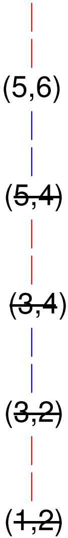


Bob now knows that his number is 6



Applications to Campaign Logic

The use of language in campaigning

After Barack Obama's comments last week about what he typically eats for dinner were criticized by Hillary Clinton as being offensive to both herself and the American voters, the number of acceptable phrases presidential candidates can now say is officially down to four. "At the beginning of 2007 there were 38 things candidates could mention in public that wouldn't be considered damaging to their campaigns, but now they are mostly limited to 'Thank you all for coming,' and 'God bless America'" [said] George Stephanopoulos.

The Onion 1 May, 2008

US presidential race

Stage 1, primaries

Candidates: Hillary Clinton, Barack Obama, John McCain

Primary Winners: Obama (Democrat), McCain (Republican)

Stage 2, presidential election

Presidential winner: Barack Obama

Another illustrative example

- Hillary Clinton (while campaigning in Indiana):

  $D =$ `As a child, I shot a duck.`
- Why would she say that?
  - Indiana is a conservative state. So most of her immediate audience V_1 will be conservatives.
  - Conservatives tend to **dis**favor gun control.
  - Hearing D is likely to improve HC in the eyes of V_1
    – say by amount u_1.
  - But (virtually) all statements a candidate makes are **public announcements**.
  - So another group of voters V_2 (say liberals in Massachusetts) also hear HC say D.
  - This is likely to make her go down for V_2
    – say by amount u_2.
  - But we likely have $|u_1| > |u_2|$ since
    i) $|V_1| > |V_2|$, or at least V_1 cares more passionately about the issue than V_2.
    ii) D merely **implicates** that HC will not impose gun control.

Towards a formal model: languages and theories

- We begin by considering a single candidate C.
- C's views about the issues are formulated in a proposition language $\mathcal{L}$ containing **finitely many** atomic propositions $At = \{P_1, \ldots, P_n\}$.
- For instance:
  - $P_1 =$ `We should withdraw from Iraq.`
  - $P_2 =$ `I will impose no new taxes.`
  - $\ldots$
  - $P_n =$ `We should bail out the banks.`
- $T_a = C$'s **actual theory** (i.e. the entirety of her views)
- $T_c = C$'s **current theory** (i.e. what's she's said thus far)
- Typically (but not always) $T_c \subseteq T_a$.

Worlds and preferences

- We conflate propositional valuations and worlds $w \in 2^{At}$.
- We also define $w[i] = \begin{cases} 1 & w \models P_i \\ -1 & w \not\models P_i \end{cases}$
- We initially consider a single group of voters V (think of this as a constituency).
- The voters in V are characterized by their preference for a set of ***ideal worlds***.
- This is formalized via two functions p_v, x_v:
  - $p_v(i) = \begin{cases} 1 & V \text{ would prefer } P_i \text{ to be true} \\ 0 & V \text{ is neutral about } P_i \\ -1 & V \text{ would prefer } P_i \text{ to be false} \end{cases}$
  - $x_v : At \rightarrow [0, 1]$ the weight which V assigns to P_i s.t. $\sum w_v(i) \leq 1$.

Utilities of worlds and theories

- The utility of a world for V is defined as

$$u(w) = \sum_{1 \le i \le n} p_v(i) \cdot x_v(i) \cdot w[i]$$

- Note that a candidate's current theory T_c is likely to be **incomplete** – i.e. she may not express a view on some P_i.
- To calculate the utility of an arbitrary T we need to know how V will "fill in the blanks."

Voter types

- We claim that there are least three types of voters:
  - **Optimistic voters** (assume the best about C given T_c)
  - **Pessimistic voters** (asssume the worst about C given T_c)
  - **Expected value voters** (average across possibilities compatible with T_c).

Voter types

- optimistic voters: $ut^o(T) = \max\{u(w) : w \models T\}$
- pessimistic voters: $ut^p(T) = \min\{u(w) : w \models T\}$
- expected value voters: $ut^e(T) = \frac{\sum_{w \models T} u(w)}{|\{w : w \models T\}|}$

The value of a message

- Suppose T is the logical closure C of T_c .
- What's the best thing for her to say next?
- Roughly: $val(A, T) = ut(T \circ A) - ut(T)$
- But the precise definition will depend on
  - the kind of voter we're assuming (i.e. **o** vs. **p** vs. **e**)
  - the set from which A is selected
- Wrt the latter, consider A from
  - $\mathcal{X}_a = T_a$ (i.e. only "true convictions")
  - $\mathcal{X}_t = \mathcal{L} - \{\neg A : T_a \vdash A\}$ (i.e. anything consistent with "true convictions" = **tactical**)
  - $\mathcal{X}_m = \mathcal{L} - \{\neg A : T_c \vdash A\}$ (i.e. anything consistent with the current theory = **Machiavellian**)
  - $\mathcal{X}_\ell = \mathcal{L}$ (i.e. any sentence in the language, allowing for contradictions and lying)
- Note: $\mathcal{X}_a \subseteq \mathcal{X}_t \subseteq \mathcal{X}_m \subseteq \mathcal{X}_\ell$

The value of a message (cont.)

- If we have $\mathcal{X} = \mathcal{X}_\ell$ then T_c may become **inconsistent.**
- In this case, $\circ = *$ (i.e. an AGM-like update operation).
- In the other cases, $\circ = \dot{+}$ (logical closure).
- If $\mathcal{X} = \mathcal{X}_a, \mathcal{X}_t$ or $\mathcal{X}_m$, then we let

$$val(A, T) = ut(T \dot{+} A) - ut(T)$$

  where ut is one of ut^o, ut^p or ut^e.
- We can now define **best statements** for C given T from $\mathcal{X}$ as follows:

$$best(T, \mathcal{X}) = argmax_A val(A, T) : A \in \mathcal{X}$$

Informal example

- Candidate = McCain
- Consider the statements
  - *A* = `We should pull out from Iraq.`
  - *B* = `We must achieve victory in Iraq.`
  - *C* = `We should stay but change strategy.`
  - *D* = `We should maintain Bush's strategy.`
- Suppose
  - $best(\mathcal{X}_\ell, \emptyset) = A$ (i.e. *ceteris paribus*, *A* is most valuable)
  - McCain has already said *B* – i.e. $B \in T_c$.
  - $T_c \vdash B \rightarrow \neg A$ and hence $T_c \vdash \neg B$.
  - McCain has chosen to be Machiavellian – i.e. $\mathcal{X} = \mathcal{X}_m$.
- It could happen that $best(T_c, \mathcal{X}_m) = C$
  even though $D \in T_a$.

Single voter

- Suppose $T_c = \{P_1 \vee P_2, P_1 \rightarrow P_3, P_2 \rightarrow \neg P_3\}$
- There are two assignments satisfying T_c:
  $w = \langle 1, 0, 1 \rangle$, $w' = \langle 0, 1, 1 \rangle$.
- Consider a single voter V_1 with the following preferences:
  - $p_1(1) = 1, p_1(2) = 0, p_1(3) = 0$
  - $x_1(1) = .5, x_1(2) = 0, x_1(3) = 0$
- What should C say?
- (Note: she only needs to consider P_1, P_2.)
- $ut_1^e(T) = \frac{\sum_{w \models T} u_1(w)}{|\{w : w \models T\}|} = (.5 + -.5)/2 = 0$
- $ut_1^e(T \dot{+} P_1) = \frac{\sum_{w \models T \dot{+} P_1} u(w)}{|\{w : w \models T \dot{+} P_1\}|} = .5/1 = .5$
- $ut_1^e(T \dot{+} P_2) = \frac{\sum_{w \models T \dot{+} P_2} u(w)}{|\{w : w \models T \dot{+} P_\}|} = -.5/1 = -.5$
- So $best_1(T_c, \mathcal{X}_m) = P_1$.

Multiple voters

- Consider a second voter V_2 with the following preferences:
  - $p_2(1) = -1, p_2(2) = 1, p_2(3) = 1$
  - $x_2(1) = .5, x_2(2) = .25, x_2(3) = 0$
- $ut_2^e(T) = \frac{\sum_{w \models T} u(w)}{|\{w : w \models T\}|} = (-.75 + .75)/2 = 0$
- $ut_2^e(T \dotplus P_1) = \frac{\sum_{w \models T \dotplus P_1} u(w)}{|\{w : w \models T \dotplus P_1\}|} = -.75/1 = -.75$
- $ut_2^e(T \dotplus P_2) = \frac{\sum_{w \models T \dotplus P_2} u(w)}{|\{w : w \models T \dotplus P_2\}|} = .75/1 = .75$
- So $best_2(T_c, \mathcal{X}_t) = P_2$.
- So if V_1 and V_2 are both in the audience, C should say P_2.
- In general, $Best_{\mathcal{V}}(T, \mathcal{X}) = argmax_A \sum_{i \in \mathcal{V}} val_i(A, T) : A \in \mathcal{X}$.

Complex statements

Proposition (1)

Assume **e**-voters. For all A, B s.t. $A, B, A \wedge B \in \mathcal{X}_m$, (i.e., $A, B, A \wedge B$ consistent with T_c) there exist $a, ..., f \in [0, 1]$ s.t.

1) $a \cdot val(A, T) + b \cdot val(\neg A, T) = 0$
2) $val(A \wedge B, T) = val(A, T) + val(B, T \dotplus A) = val(B, T) + val(A, T \dotplus B)$
3) $c \cdot val(A \vee B) + d \cdot val(A \wedge B, T) = e \cdot val(A, T) + f \cdot val(B, T)$

Proof: For 1), $ut(T) = a \cdot ut(T + A) + (1 - a) \cdot ut(T + \neg A)$ where $a = \frac{\{w \mid w \models T \dotplus A\}}{\{w \mid w \models T\}}$.

Moving to complete theories

Corollary

*There is a **complete** $T \supseteq T_c$ s.t. $ut^e(T) \geq ut^e(T_c)$.*

Proof: From the above, we must have exactly one of

i) $val(P_i, T) = val(\neg P_i, T) = 0$

ii) $val(P_i, T) > 0$ and $val(\neg P_i, T) < 0$

iii) $val(P_i, T) < 0$ and $val(\neg P_i, T) > 0$

Suppose $Q_i, \ldots, Q_k$ ($k \leq n$) are all the atoms not in T_c.

Let $T_0 = T_c$ and $T_{i+1} = \begin{cases} T_i \cup Q_i & val(Q_i, T_i) \geq 0 \\ T_i \cup \neg Q_i & \text{else} \end{cases}$

Let $T = Cn(T_k)$.

Moving to complete theories (cont.)

Corollary

One of the best extensions of T_c is a complete theory $T \supseteq T_c$

Proof:

- Suppose T' is a best extension of T_c and T' is incomplete.
- By the previous corollary, there is $T'' \supseteq T'$ which is a complete extension of T' (and thus of T_c) such that $ut^e(T'') \geq ut^e(T')$.

Moving to complete theories (cont.)

- The previous result suggests that if *C* assumes **e**-voters, then it will never be to *C*'s disadvantage to move towards a complete theory.
- But why then do we have the *Onion* phenomenon?
- I.e. why do candidates state vacuities like "`God bless America`" or "`9/11 was a tragedy`."
- Conjecture: They must be assuming that there are at least some **o**-voters (who 'always assume the best').
- $T \supseteq T' \Longrightarrow \max\{u(w) \mid w \models T'\} \leq \max\{u(w) \mid w \models T\}$
- I.e. $T \supseteq T' \Longrightarrow ut^o(T') \leq ut^o(T)$

Does order matter?

- Does the order in which C says A and B matter?
- Proposition 1.2 suggests "no" in the case A, B are consistent with T_c.
- This may seem like a counter-intuitive result:
  - $A =$ `Read my lips: 'no new taxes.'`
  - $B =$ `We must institute user fees.`
- ***A***; ***B*** allowed Bush senior to seem as if he favored low taxes and small government.
- ***B***; ***A*** might have had the opposite result.
- Our current model doesn't account for this.
- Planned extensions
  - extend with a formal model of implicature
  - type dynamics: after hearing A, maybe some voters **change type** from expected value to optimistic

Independent topics

- Suppose that A and B are in disjoint languages (and hence about unrelated topics).
- e.g. $A \in \mathcal{L}_1$ is about abortion, $B \in \mathcal{L}_2$ is about Iraq.
- Order doesn't seem to matter in this case.
- Then we should have

$$val(A, T) + val(B, T * A) = val(B, T) + val(A, T * B)$$

even if any of $A, B, A \wedge B$ are **inconsistent** with T_c.

- I.e. even if $T_c \vdash \neg A$, then updating $A; B$ should have the same effect as update $B; A$.
- The next result addresses this point ...

Independent topics (cont.)

Definition

Let T be a theory in the language $\mathcal{L}$, $\langle \mathcal{L}_1, \mathcal{L}_2 \rangle$ a partition of $\mathcal{L}$ into disjoint sublanguages.

- We say that $\mathcal{L}_1, \mathcal{L}_2$ **split** T if there are $A \in \mathcal{L}_1$, $B \in \mathcal{L}_2$ s.t. $T = Cn(A, B)$.
- Similarly we say that pairwise disjoint languages $\mathcal{L}_1, .., \mathcal{L}_n$ split T if there are $A_i \in \mathcal{L}_i$ s.t. $T = Cn(A_1, ..., A_n)$.
- In such a case, we say that $\langle L_1, ..., L_n \rangle$ is a T**-splitting**.

Independent topics (cont.)

Proposition

(Parikh '99, Kourousias and Makinson 2007) Every first order theory has a unique finest splitting.

Thus a theory can be seen uniquely as consisting of a number of subtheories, each about its own subject matter. A numerical notion of information can be defined for the propositional case (Parikh '09) and it can be shown that T splits into T_1 and T_2 iff the information in T is no more than the information in T_1 plus T_2.

Independent topics (cont.)

Example: Suppose T is generated by the two axioms, $P, Q \vee R$. Then T splits into T_1 generated in the sublanguage $\{P\}$ by P, and T_2 generated in the sublanguage $\{Q, R\}$ by $Q \vee R$. T cannot be split further.

Independent topics (cont.)

Proposition

Suppose

- C's current theory is T over language $\mathcal{L}$.
- $\mathcal{L}$ can be split into $\mathcal{L}_1, \mathcal{L}_2$.
- Let $A \in \mathcal{L}_1$ and $B \in \mathcal{L}_2$ be **any** statements that the candidate could make.

Then $val(A, T) + val(B * A) = val(B, T) + val(A, T * B)$ where $*$ is an update operator satisfying $T * A * B = T * B * A$.

Other Considerations

Motorist: My car is out of gasoline.
Passerby: There is a gasoline station around the corner.

The passerby has not *said* but has *implicated* that as far as she knows, the gasoline station is open. As Savage and Austin have pointed out, each statement made is also an *action* and is evaluated as a move in a game. In this case, the game is cooperative as the passerby (presumably) only wants to help the motorist.

In other situations, there could be an element of opposition between a speaker and a listener, and the listener will learn to read *between the lines.*

A candidate who is aware of the fact that her words are being *interpreted* will speak in awareness of this fact.

Future Work

- candidates address multiple groups of voters with partial knowledge of their relative sizes
- multiple candidates (their statements can interact and they can speak about or reply to each other)
- outside events (i.e. "nature" sequentially makes certain propositions true with probabilities either known or unknown to the candidates – e.g. hurricanes, bank failures)
- enriching the language used by the candidates
- e.g. with a conditional operator to formalize

  `If Israel attacks Iran, then the US must ...`
- extend formal theory of implicature [after Jaeger or Parikh & Ramanjuan]