6 Voting theory

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Abstract The theory of voting has a long and discontinuous history. Currently the theory takes individual preference rankings over alternatives as given and establishes results involving compatibility or incompatibility of various desiderata concerning ways to aggregate individual rankings into social choices or rankings. From the viewpoint of democracy some of those desiderata are more important than others. We review some of the relevant results in this area. Since the main results are of negative nature, it makes sense to ask whether those results could be avoided if some other view were to be adopted regarding the form of individual opinions.

1 Introduction

Although it is true that voting is not a sufficient condition for democratic governance, it is certainly a necessary condition thereof. Indeed, along with bargaining it belongs to the most important ways of reaching collectively binding decisions. Voting is resorted in a wide variety of contexts: political elections, decision making in multi-member bodies, electing best entries in song contests, determining the winners in figure-skating, issuing verdicts in juries, electing officers to various positions in public organizations etc. Voting is sometimes used in purely informal and ad hoc settings, such as when a group of people is deciding how to spend an evening together, or a family is deciding on the name of a just acquired pet.

Perhaps as a consequence of the variety of contexts in which voting is resorted to, it comes in many different forms or procedures. The most common in informal contexts is the plurality voting or one-person-one-vote procedure. It imposes minimal requirements on the voter: he/she (hereinafter she) should only indicate one alternative that she considers superior to others. Whatever else information she might be able to provide regarding her opinion of the alternatives is simply not used.

In 1770 Jean-Charles de Borda gave a presentation in the French Royal Academy of Sciences pointing out that the plurality voting system may lead to a choice of a highly implausible alternative, viz. one that would be defeated by all other contestants by a pair-wise vote by a majority of voters (McLean & Urken 1995). For this to happen, the voters would need to have preferences over all pairs of alternatives or a preference ranking over them.

Borda's observation and the accompanying suggestion for a voting system – today known as the Borda Count – received a fair amount of attention in the pre-revolution France and in the immediate aftermath. Borda suggested that the winners be elected on the basis of their positions in the voters' preference rankings. Each rank would give an alternative a fixed number of points so that the lowest rank would give a points, the next to lowest a+b points, the next a+2b points, etc. The points given to each alternative would then be summed up to a (Borda) score and the alternative with the highest score declared the winner. The essential feature of the system that Borda called the method of marks is that the point difference between any two consecutive ranks is a constant b. This system is today known as the Borda Count. In practice, the values a=0 and b=1 are used. What Borda also showed was that this system can be implemented on the basis of pair-wise votes: by summing the number of votes an alternative gets in each pair-wise vote against other alternatives one gets the Borda score of the alternative.

Borda's contemporary Marquis de Condorcet became the first critic of Borda's proposal and an advocate of systems based on pair-wise comparisons. Condorcet showed that the Borda Count may not elect a candidate that would defeat all other candidates in pair-wise contests by a simple majority. Today such a candidate is called the Condorcet winner. The tension between two types of notions of winning was thereby created. One emphasizes the positions that alternatives occupy in voters' preference rankings: the higher on the average, the better. This view is called positional. The other stresses the pair-wise victories of alternatives: the more alternatives are defeated by a given alternative, the better are the chances of the latter's becoming the overall winner. This tension between two notions of winning is still visible in the contemporary literature.

After the times of Borda and Condorcet there was a period of more than half a century over which no significant contributions were made to the theory of voting. In the latter half of the 19th century two names should be men-

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tioned. C.L. Dodgson, an Oxford mathematics tutor, wrote several pamphlets on voting systems i.a. apparently re-inventing the Borda Count (see Black 1958). The second name from roughly the same period is E. J. Nanson. He invented a multi-stage elimination method for reconciling the positional and pair-wise winning notions (Nanson 1882).
Nanson’s contribution was followed by yet another period of more than half a century over which no major contributions were made to the theory of voting procedures. The “third wave” of the theory began with the work of Black and Arrow in late 1940’s and early 1950’s (Black 1948; Arrow 1951). This wave is still in full swing. The aim of this article is to provide an overview of the theory of voting: why it is important, how the problems are addressed and which are the main results. The next section gives a motivation for the study of voting. Section 3 deals with those properties of voting systems that we intuitively deem desirable. The next section reviews some important results on the compatibility relationships between these desiderata. We then focus on the “givens” of the theory and look at systems that require less of the voters than complete and transitive preference relations. Thereafter we discuss briefly some systems that boil down to aggregating cardinal utilities of voters.

2 Motivation

Voting procedures are used in many contexts. We have procedures for electing presidents, parliaments and other assemblies. We also have procedures for deciding which legislative proposal is to be adopted, which verdict to issue in court cases, and so on. In some cases the outcome of voting – the winner – is a person to occupy an office. In others, the winners constitute a group of persons. The winner may also be a proposition or policy alternative. Given the variety of settings in which voting is resorted to it is not surprising that there are many voting procedures. What is perhaps more unexpected is that there are several procedures used for what appears to be an identical purpose, e.g. electing a president. Yet there are, and, what is more important, they are not equivalent. Indeed, the procedure is an equally important determinant of the voting outcome than the voter opinions. A hypothetical example illustrates this.
Suppose that there are three candidates, Brown, Jones and Smith, running for presidency. Suppose moreover that the electorate is divided into three groups. Group 1 supports Brown and views Smith as the second best. Group 2 supports Jones and regards Smith preferable to Brown. Group 3, in turn, regards Smith the best, Jones second best and Brown the worst of candidates. Group 1 consists of 40 %, group 2 of 35 % and group 3 of 25 % of voters. The voter opinions can be presented in the following tabular form.

Group 1 (40 %): Brown Smith Jones
Group 2 (35 %): Jones Smith Brown
Group 3 (25 %): Smith Jones Brown

If this is the distribution of opinions reported by the voters in the polls, the outcome is completely dependent on the procedure used. In particular, three often used procedures lead to three different outcomes. To wit, the plurality voting elects Brown as this candidate is ranked first by the largest number of voters. Jones, in turn, would win in plurality runoff (or instant runoff) election since – as no candidate is ranked first by more than 50 % of the voters - Brown and Jones would make it to the second round where Jones beats Brown with the aid of the votes of the Smith supporters. Most pair-wise comparison methods would, in turn, end up with Smith since Smith beats both Brown and Jones in pair-wise contests by a majority of votes. Smith is thus the Condorcet winner.
So, three commonly used procedures end up with three different outcomes in this electorate. In fact, then, any candidate can be rendered the winner by varying the procedure. Is there a way of telling which procedure is “best”? This requires a study of the requirements that we impose on a good voting system and on the extent to which various procedures satisfy those requirements.

3 Procedure desiderata

In the above example the three voting procedures lead to different choices. Nonetheless these three can be seen as generalizations of a common intuitive notion of winning, viz. whichever candidate or alternative is ranked first by an absolute majority – i.e. more than 50 % - of voters should be elected. Obviously, should this kind of opinion distribution or preference profile be encountered, all three procedures would end up with the same winner. A candidate ranked first by an absolute majority of voters is called the strong Condorcet winner. The requirement that eventual strong Condorcet winners should always be elected can be called the strong Condorcet winner criterion. It is satisfied by a large class of voting procedures. Not by all, though. Consider the following 13-voter profile over 3 candidates A, B and C:

8 voters: A B C
5 voters: B C A

Obviously A is now the strong Condorcet winner and would thus be elected by the above three voting procedures. Assume, however, that each voters is allowed to vote for two candidates and that the winner is the candidate receiving more votes than any other candidate. This system would elect B in this example. So would the approval voting which is a system that allows the voters to vote for as many candidates they like so that each voter can give each candidate either one or zero votes (Brams and Fishburn 1983). The winner is the candidate that has more votes ("approvals") than the others. If we assume that the 8 voters approve of both A and B and the 5 voters either B or B and C, the winner is B. The same outcome ensues from the Borda Count. It can be seen that the two notions of winning advocated by Condorcet, on the one hand, and Borda, on the other, are indeed quite incompatible. Even in highly consensual societies these two requirements may come up with different outcomes. The worse for the Borda Count and approval voting, one could say.
This is not necessarily a correct conclusion, however. Although a strong Condorcet winner is quite robust with regard to eliminating alternatives, it is not robust at all with respect to adding or subtracting voters with opinions that intuitively should make no difference to the outcomes (Saari 1995). To illustrate, consider again the above 13-person voting body and assume that it is expanded by adding 15-voters with the following opinions of the three candidates A, B and C:

5 voters: A B C
5 voters: B C A
5 voters: C A B

The added group of 15 voters is obviously quite incapable of making any difference at all between the three candidates unless some voters' opinions are given more weight than the others'. So, this group should – intuitively speaking – have no impact on the outcome of voting. But it does if the pair-wise winners determine the outcome. It turns out that after the expansion of the voting body into a 28-member one, A is no more the Condorcet winner. Now it is B. This kind of "instability" casts a shadow over the Condorcet winning criterion. At the same time it should be pointed out that the Borda Count winners are vulnerable to changes in the candidate set: if a candidate is removed, the Borda Count winner may change (Fishburn 1974). Indeed, the ranking determined by the Borda scores may be completely reversed upon adding or subtracting candidates.
Certain kind of stability is, indeed, often regarded as a desirable property of voting rules. The voting outcomes should remain invariant under specific changes in the voting situation. For example, in democratic voting rules, one typically expects that the order in which ballots are cast or re-naming of the voters should make no difference in the voting outcomes. It is the distribution of opinions, not who has a given opinion that should determine the outcome. This often implicitly assumed voting system desideratum is called anonymity. A similar requirement states that a re-labelling of the decision alternatives – e.g. re-naming the candidates – should make no difference to the content of the voting outcomes. After the re-naming the same alternatives should emerge as winners, albeit carrying new labels. This requirement is known as neutrality: the procedures should treat alternatives in a neutral way.
Despite their self-evident nature the requirements of anonymity and neutrality are not always satisfied even settings that are generally deemed democratic. In many voting bodies the vote of the chairperson has a special significance when the opinions of the members are evenly distributed, i.e. when there is a tie. In those special circumstances, the chairperson's vote is often used as the tie-breaker. Thus, the system is not anonymous.

Similarly, in legislative settings, the status quo alternative often has a special status. In the widely used amendment system the legislative proposals and amendments are voted upon in pairs so that the winner of each pair confronts the next proposal according to a specific agenda. Of k proposals k-1 pairwise votes are taken and the winner of the last one is the overall winner. In this system the agenda plays a crucial role in determining the outcome. To illustrate, consider again the preceding 15-voter profile over alternatives A, B and C. Suppose that the agenda of pair-wise comparisons is: 1. A vs. B, and 2. the winner vs. C. Call this agenda I. If each voter votes according to her preferences in both ballots, the winner is C. Now, if the agenda were: 1. B vs. C, and 2. the winner vs. A, the winner would be A under the same assumptions. Let us call the latter agenda II. So, the agenda is decisive under what is called sincere voting, i.e. if all voters reveal their true preferences at each stage of the voting process.

The agenda-dependence of the outcomes does not, however, hinge on the sincere voting assumption. Suppose, instead, that the voters are sophisticated in the sense of resorting to backwards induction in determining their choices at each stage (McKelvey and Niemi 1978). This means that under agenda I the voters first determine who to vote in stage 2. In that stage C is confronted with either A or B depending on the outcome of stage 1. Since stage 2 is the last one, the voters can be assumed to vote according to their preferences. Hence if it is A that faces C in stage 2 the winner is C, while if it is B that is compared with C, the winner is B. Thus, in effect the choice that the (sophisticated) voters are faced with in stage 1 is between B and C even though the agenda says that A and B are being compared. Since the majority prefers B to C, the outcome is B under sophisticated voting. Consider now agenda II. By the same argument, the winner under sophisticated voting is C.

The agenda-dependence of the amendment system makes it non-neutral. In legislative settings it is typically the status quo that confronts whichever alternative survives the preceding stages of the agenda. Under sincere voting it thus has a favourable position with respect to other alternatives. Under sophisticated voting, in turn, it seems to have a disadvantage. In any event the conclusion is that the amendment procedure does not treat alternatives in a neutral fashion.

Another type of stability requirement pertains to the very rationale of voting, viz. the idea that by voting one affects outcomes in a "natural" way. Expressing support for an alternative should increase or at least not decrease its probability of being chosen. More precisely this requirement can be stated in two non-equivalent ways (Nurmi 1999; Campbell and Kelly 2002). (i) Given a preference profile, it should never be harmful for an alternative if some voters rank it higher than in the profile, ceteris paribus. (ii) Given a preference profile, it should never benefit a voter to abstain, i.e. the outcome resulting from abstaining should never be better in the voter's opinion than the one resulting from her voting according to her preferences. Requirement (i) is known as monotonicity and requirement (ii) as invulnerability to the no-show paradox or participation axiom. The former is an intra-profile property, that is, it considers a given profile of, say, n voters and k alternatives. The latter, in turn, is an inter-profile requirement: it considers two profiles with different number of voters. Yet, they are pretty close to each other. Monotonicity deals with choice sets resulting from the voters' changing their mind with respect to an alternative vis-à-vis the others, while requirement (ii) considers changes in voting outcomes resulting from eliminating preference rankings from the profile. Despite their intuitive appeal, they are violated by some commonly used voting systems.

Consider the following profile:

47 voters: A B C
2 voters: A B C
25 voters: B C A
26 voters: C B A

Suppose that the plurality runoff system or instant runoff (IRV) is being used. Obviously, A and C will get to the second round, while B is eliminated. In the second round C defeats A by 51 votes to 49. Subtract now the 47 voters mentioned first (nearly a half of the electorate) and conduct a new election for the remaining profile. Now B and C make it to the second round, where B wins. Obviously, the outcome is better from the abstainers' point of view in the reduced profile than in the original one. Hence the plurality runoff system fails on the participation axiom or, in other words, is vulnerable to the no-show paradox, i.e. not voting may lead to a better outcome than voting.

The single transferable vote (STV) system is also vulnerable to the no-show paradox. This can be seen from the same example since with three alternatives the plurality runoff system and STV are equivalent: eliminating the

alternative with the smallest number of first place votes is tantamount to qualifying the two most vote-getters to the second round.
The non-monotonicity of IRV and STV is shown in the following example:

35 voters: A B C
33 voters: B C A
32 voters: C A B

Here the second-round contestants are A and B, whereupon A wins. Suppose now that 2 -14 of those voters with the preference ranking B C A would rank A first, ceteris paribus. Then the runoff contestants would be between A and C with C winning the overall contest. Hence additional support turns the winner (A) into a non-winner.
In the interpretation adopted above, monotonicity requires that additional support, ceteris paribus, is not to render winners into non-winners. In the mechanism design literature one encounters another related concept, viz. that of Maskin monotonicity (Maskin 1985). It states that if an alternative x is a winner in a profile P and if a profile Q is constructed so that x's position remains the same or improves with respect to all other alternatives, then x is the winner in Q as well. Note that no ceteris paribus condition is imposed on other alternatives vis-à-vis each other. It turns out that Maskin monotonicity is a very strong desideratum: none of the voting systems discussed above satisfies it.
In some voting systems one may encounter a strong version of the no-show paradox. To wit, it may happen that a group of voters with identical preferences may succeed in getting their favourite (i.e. first-ranked) alternative elected by abstaining, while some lower-ranked alternative would win if they vote according to their preferences (Felsenthal 2001). To illustrate the paradox consider the following 19-person profile and Nanson's method (Nurmi 2005, 34):

6 voters: c a d b
5 voters: a b d c
5 voters: b c d a
1 voter: c b a d
2 voters: c b d a

Nanson's method ends up with b in this profile. If the last mentioned 2 voters abstain, the choice is C, their favourite.
Monotonicity and invulnerability to the no-show paradox are by no means the only stability desiderata imposed on voting systems. An intra-profile criterion known as consistency requires that coinciding choices made by subelectorates be preserved in the choices made by the electorate at large. More precisely, for a system to be consistent, the following has to hold for any two mutually excusive and jointly exhaustive subsets of voters: if using the same procedure the subsets end up with at least partially overlapping choices, then those alternatives chosen by both subgroups are also chosen by the electorate at large, i.e. by the union of those subgroups.
This intuitively plausible desideratum turns out to be relatively uncommon among voting systems. For example the plurality runoff system is inconsistent as shown by the following example. The electorate consists of two profiles one of which is:

3 voters: A B C
3 voters: B C A
2 voters: C A B

The other is:

3 voters: A C B
2 voters: C B A

In both profiles A is elected by the plurality runoff, IRV and STV. However, in the combined 13-voter profile the winner is C no matter which of these systems is resorted to.

In contradistinction to these three (and many other) systems, the plurality voting, Borda Count as well as approval voting are all consistent. The same is obviously true of vote-for-two or, in general, vote-for-k systems in which each voter is to vote for a fixed number of alternatives and the winner is the one with a larger vote sum than any other alternative. Consistency of these systems offsets at least to some extent the fact that they all fail on the Condorcet winning criterion, i.e. they all may fail to elect a Condorcet winner.

Somewhat less common stability condition is called the Chernoff property or property alpha (Chernoff 1954). It states that if an alternative is the winner in a set of alternatives, it should be the winner in every proper subset of those alternatives it belongs to, ceteris paribus. Despite its intuitive appeal, this condition is very uncommon among known voting systems. In fact, of those discussed above only the approval voting satisfies it under the assumption that ceteris paribus means that not only the preferences of voters but also their approved alternatives remain the same in each subset considered.

In addition to desiderata pertaining to stability of voting outcomes under various modifications of profiles or alternative sets, there is a property that captures the intuitive notion that under sufficiently large modifications of the preferences, the choice sets should change as well. More specifically, if all voters turn their preferences upside down, i.e. for each pair of alternatives they reverse their opinion, then the outcome of voting should change as well. We say that a procedure exhibits reversal bias if there is a preference profile so that when every voter switches her preference between each pair of alternatives, the voting outcome remains the same as before the change. It is easy to construct examples of profiles where both plurality voting and the plurality runoff and hence IRV exhibit reversal bias (see Nurmi 2005, Saari and Barney 2003).

The desiderata of voting systems are many and, thus, we have touched upon only a small subset of them (see Fishburn 1977, Nurmi 1987, Richelson 1979, Straffin 1980, Smith 2000). Rather than defining voting systems and desirable criteria, a more efficient way to look for good ways to aggregate opinions is to look at the compatibility and incompatibility of various desiderata. In the following section we shall review some of the most important results of this type of work.

4 Some important results

By far the best-known result in the theory of social choice is Arrow's theorem. It is not only important substantially, i.e. in saying what it says about social choice, but also methodologically. Arrow (1951; 1963) set the stage for a host of results similar in spirit in showing that certain sets of voting system desiderata are unachievable because the properties included in those sets are mutually incompatible. Arrow's theorem deals with social welfare functions which are mappings from Cartesian products of individual preference relations into similar collective preference relations. By similar we mean that both preference relations are binary, complete and transitive over the set of alternatives. Arrow, thus, assumes neither more nor less than individual preference rankings. In addition to this formal requirement, the following four conditions are imposed on social welfare functions:

(i) universal domain, (ii) independence of irrelevant alternatives (IIA), (iii) Pareto condition and (iv) nondictatorship. The theorem says that these four conditions together with the formal requirement that both individual and collective preference relations be complete and transitive are incompatible. Obviously, the significance and practical importance of the result depends on how important and plausible the conditions are deemed.

Condition (i) states that the function should make no restrictions with respect to allowable individual preference rankings. This sounds like a reasonable or, at least, very convenient condition. On the other hand, it implies that the likelihood of condition violations plays no role in the theorem: systems which fail on some criterion under very specific and unlikely circumstances are on par with those where one can expect a criterion violation all the time. Condition (ii) is perhaps the most controversial one of all. It states that the collective preference order between any two alternatives, say x and y, depends on the individual preference between these two alternatives only. In other words, whether x is collectively preferred to y or vice versa or both, depends only on the way the individuals rank x and y. Condition (iii) says that if each individual strictly prefers x to y, then y is not preferred to x in the collective preference relation. Finally, condition (iv) excludes dictators by saying that there be no individual whose preference relation over all each pair of alternatives coincides with the collective preference relation with respect to this pair.

All systems discussed in this article fail on IIA. Consider for example plurality voting and the first example in section 2. The collective ranking resulting from plurality system is: Brown Jones Smith. So, the ranking between Brown and Jones is such that the former is preferred to the latter. Now, consider the subset consisting of Brown and Jones. In this subset Jones is preferred to Brown. Hence, IIA is violated.

There are systems that fail on several Arrow conditions. One such system is the amendment procedure. It is well-known that it may result in a collective preference cycle: x preferred to y, y preferred to z and z preferred x etc. It thus violates the condition that the social preference relation be transitive. But it also fails on condition (iii). Consider the following profile.

1 voter: A B D C
1 voter: B D C A
1 voter: D C A B

With sincere voting and agenda 1. B vs. D, 2. the winner vs. A and 3. the winner vs. C, C wins. Yet, D is preferred to C by each voter. Thus, the amendment system fails on Pareto criterion.

Nearly equally celebrated as Arrow's is the theorem independently proven by Gibbard (1973) and Satterthwaite (1975). It deals with voting strategies and single-valued social choice functions. The latter are functions mapping all alternative sets and preference profiles into singleton sets of alternatives. These functions are also known as social decision functions. They, thus, specify for each alternative set and preference profile a single alternative, the winner. Voting strategy, in turn, indicates the preference ranking a voter reports when voting. This may be identical with her preference ranking over alternatives, but it may also differ from it. In the latter case the voter is said to misrepresent her preferences. It is plausible to assume that a voter misrepresents her preferences if the outcome resulting from misrepresentation, ceteris paribus, is preferred by the voter to the outcome resulting from her sincere voting, again ceteris paribus.

Now, a voting system is manipulable in a voting situation, i.e. in a set of alternatives and a preference profile over those alternatives, if in that situation there is at least one voter who achieves a better outcome (from her own point of view) by misrepresenting her preferences than by voting sincerely. A voting system, in turn, is defined to be manipulable if there is at least one situation in which the system is manipulable. In other words, a voting system is manipulable if the sincere voting strategies by all voters do not always lead to a Nash equilibrium.

The Gibbard-Satterthwaite theorem says that every anonymous, neutral and non-trivial social decision function is either manipulable or dictatorial. A social decision function is non-trivial if for each alternative one can construct a preference profile so that this alternative will be chosen by the system.

Prima facie, the Gibbard-Satterthwaite theorem is quite dramatic; manipulability or dictatorship do not look attractive alternatives to choose from.[2] On closer inspection it is evident, however, that this is not a doomsday message for democratic institutions. It is possible that manipulability, i.e. gaining benefit from preference misrepresentation, materializes in very rare situations only. Moreover, to benefit from preference misrepresentation the voter needs to know basically everything about the preference profile which may be a tall order in most voting bodies. Finally, the theorem deals with singleton-valued choice functions, while most voting systems may result in a tie between two or more alternatives. These remarks are not intended to play down the importance of the theorem as a theoretical result. It is certainly of great significance in pointing out that the behavioural assumptions underlying voting behaviour should be taken into account in voting system evaluations. Results, such as those cited in the preceding, on properties of voting systems that hold under sincere voting assumption may fail under sophisticated voting assumption.

The Gibbard-Satterthwaite theorem amounts to the incompatibility of non-dictatorship and non-manipulability among single-valued choice functions. Slightly later Gärdenfors (1976) proved a theorem that deals with (possibly multiple valued) choice functions or social choice correspondences. He showed that all anonymous and neutral social choice functions that satisfy the Condorcet winner criterion are manipulable. Since the Condorcet winner criterion is often regarded as a highly desirable property, this result is of the same negative type as Arrow's. The Gärdenfors theorem leaves open the manipulability of those systems that may fail to elect the Condorcet winner when one exists. Yet, it is fairly straight-forward to show that all those systems discussed above are manipulable. To show that the plurality runoff, IRV and STV are manipulable, consider the following profile of 8 voters:

[2] This is an exclusive "or" we are dealing with here since obviously dictatorial decision functions are not manipulable: the dictator can only lose by misrepresentation and the strategies of other voters are irrelevant for the outcome.

3 voters: A B C
3 voters: B C A
2 voters: C A B

With sincere voting, there is a runoff between A and B whereupon A wins. This is the least preferred alternative of the 3 voters in the middle of the profile. If one, two or all of them had voted as if their preference ranking is C B A, ceteris paribus, C would have won on the first round or after a second round against A. In any event, the outcome would have been better for the voters deviating from their true preferences in their voting strategies. The same profile can also be used in showing that the plurality voting system is manipulable. With sincere voting, the outcome is a tie between A and B. This can be broken in A's favour by one of the 2 last mentioned voters if she votes as if her first ranked alternative is A. Hence, this voter can bring about a preferable outcome by preference misrepresentation. The same profile can be used to show that sincere voting strategies do not lead to a Nash equilibrium under Borda Count, either. If all voters reveal their true preferences, the outcome is B, the lowest-ranked alternative for the 2 voters in the profile. If these voters rank A first, ceteris paribus, the outcome is A, their second-ranked alternative in their true preference ranking.

The no-show paradox is, of course, an unpleasant surprise not only for its "victims", i.e. people who would have been better off by abstaining than by voting, but also to the advocates of democratic forms of decision making. It undermines the very rationale of those forms. Therefore, Moulin's (1988) theorem which states that the Condorcet winner criterion and invulnerability to the no-show paradox are incompatible is bad news for those who deem the criterion of utmost importance. It is worth observing that the theorem does not say anything at all about systems that do not satisfy the Condorcer winner criterion. Among those there are systems that are invulnerable to the no-show paradox and those that aren't. In the first group there are systems such as plurality voting, vote for k and the Borda Count, in the latter plurality runoff, IRV and STV.

There is a stronger version of the no-show paradox which occurs when a group of voters by abstaining helps to bring about the election of their first-ranked alternative, while by participating, ceteris paribus, they would contribute to the election of some other, i.e. lower-ranked, alternative. Moulin's result leaves open the possibility that invulnerability to this stronger version would not be incompatible with the Condorcet winner criterion. These hopes are largely squashed by the theorem of Pérez (2001) which states that nearly all systems that elect a Condorcet winner when they exist can exhibit the strong version of the no-show paradox. Note again, however, that this theorem does not extend to systems that fail on the Condorcet winner criterion. Plurality runoff system appears to be invulnerable to the strong version of the paradox. By abstaining a group of voters may, if anything, block the entry of their favourite to the second round, but not increase its chances of being elected. Similarly, in STV and IRV the abstainers increase the likelihood of their favourite being eliminated.

The above is but a small and biased sample of various incompatibility results pertaining to voting systems. To counterbalance them there are several important results on compatibility of various desiderata. Perhaps the best-known of these is May's (1952) characterization of the majority rule as the only rule defined over two alternatives that satisfies (1) anonymity, (2) neutrality, (3) duality and (4) strict monotonicity. Of these conditions (1) and (2) have been touched upon above. (3) says that if each voter reverses her preference over the two alternatives, then outcome is also reversed. (4) states that if there is a tie, only one individual's preference change is needed to break it to the direction of the preference change. May's result thus states that the conditions (1)-(4) characterize the simple majority rule and, conversely, any rule that satisfies these conditions is equivalent to the simple majority rule.

Some other voting systems have also been axiomatized. One of them is the Borda Count. Young (1974) shows that the Borda Count is the only system that satisfies: (1) neutrality, (2) consistency, (3) faithfulness and (4) cancellation property. Faithfulness is the very natural requirement that the system be such that if the collective body consists of only one individual, then the winner according to the system coincides with her first-ranked alternative. The cancellation property, in turn, is satisfied by systems which, in a situation where for each pair of alternatives x and y the number of voters preferring x to y is equal to the number of voters preferring y to x, result in a tie between all alternatives. Young's result then states that the Borda Count has the properties (1)-(4) and conversely, any system that has these properties is equivalent to the Borda Count.

Also approval voting has been given an axiomatic characterization (Fishburn 1978). There are three axioms in this system: neutrality, consistency and disjoint equality. The last-mentioned is the requirement that if two individuals come up with two distinct choices A and B from the same set of alternatives, then when forming a collective body of the two individuals, its choice set coincides with the union of A and B.

The theory of social choice has thus both good and bad news for the designer of voting institutions. Summarizing, the bad news are that no system satisfies all conceivable desiderata, not even the most important ones. Positional methods, such as plurality voting and Borda Count, tend to perform poorly in terms of the Condorcet winner criterion, while doing very well in terms of consistency and monotonicity. The systems based on pair-wise comparisons of alternatives, in turn, do in general well in terms of the Condorcet winner criterion, but are typically inconsistent and vulnerable to the no-show paradox, most even to the strong version of it. Multi-stage systems, such as plurality runoff, IRV, Nanson and STV, are in general non-monotonic and inconsistent.

Systems Requiring Minimal Information

The theory of voting systems is an applied field of social choice theory. As such it is largely based on Arrovian assumptions about the form in which individual opinions are expressed: the individuals are endowed with complete and transitive preference relations over the alternatives. However, most of the systems of voting used in practice do not require voters to have anywhere nearly so structured preferences. In many systems the voter simply places a check or cross next to the candidate she votes for or writes down the number corresponding the candidate or party. It is true that STV and IRV are based on the expectation that the voters reveal more about their opinions than simply one candidate, alternative or party, but these systems are used in relatively few large scale elections. More often than not, singling out one alternative is what the voters are expected to do. So, how does this practical limitation go together with the theoretical assumption of complete and transitive preferences?

In principle, there is no inconsistency in assuming that the voters have preference rankings over all alternatives and devising a system that only accepts one alternative from each voter. Indeed, in the above examples of plurality voting we have throughout assumed that people have preference rankings over all alternatives, but typically vote for just their top-ranked one or – in case of strategic voting – some other alternative. The point is that the voter is assumed to have a dichotomous classification of alternatives – the best vs. the others – instead of a preference ranking. If this would be the case, then most of the negative results would go away since the crux in achieving them is that the voters be able to provide a ranking, i.e. an opinion of preference regarding also those alternatives they do not cast their vote for. For example, to find out about monotonicity violations one has to know how the voters rank not only those alternatives they vote for, but also at least some of those they do not vote for. The same argument goes for the vulnerability to the no-show paradox.

Another related issue is the impact of the existing voting system on the structure of preferences of voters. It makes sense to argue that if the system works with dichotomous preferences, the voters tend to think in a dichotomous manner about the alternatives. Why bother with preference rankings as long as the system allows you to tell the best one apart from the others only? If we are dealing with dichotomous preferences rather than preference rankings, the available voting system repertoire becomes more restricted: systems like the Borda Count, STV or IRV are not applicable. As pointed out above, many of the negative results are also thereby avoided. But is it really plausible to assume that the voters can only classify candidates into two groups: good and less than good ones?

If this assumption is made, then a natural social choice assumption to start from is that, instead of complete and transitive preference relations over the alternatives, the voters are endowed with individual choice functions. Consequently the task of voting system designer becomes to look at the properties of various choice function aggregation rules. Considerably smaller number of pages has been written on choice function aggregation than on preference ranking aggregation. One of the magna opera in this field is Aizerman and Aleskerov's (1995) study. For our purposes the main message of the book is negative: substituting individual choice functions for individual preference rankings seems to lead to analogous – albeit not identical – incompatibility results as in the mainstream literature. To illustrate, consider the following (plausibility) conditions on collective choices based on individual choice functions[3]: (i) citizen sovereignty: for any alternative x, there exists a set of individual choice function values so that x is will be elected, (ii) choice-set monotonicity: if x is elected under some profile of individual choices, then x should also be elected if more individuals include x in their individual choices, (iii) neutrality, (iv) anonymity, and (v) choice-set Pareto: if all individuals include x in their individual choice sets, then the aggregation rule includes x as well , and if no voter includes y in their individual choice set, then y is not included in the collective choice.

[3] The conditions are those of Aizerman and Aleskerov (1995, 236), but I have taken the liberty of naming them.

In social welfare functions the aim is to impose the same formal properties on the aggregation rule as on the individual opinions: completeness and transitivity of preference relations. Surely some conditions have to be imposed on choice set aggregation rules to distinguish reasonable from unreasonable ones. Consider Chernoff's condition. [4] It will be recalled that it states the following. If an alternative is among winners in a large set of alternatives, it should also be among the winners in every subset it belongs to. Another similar property is concordance. Suppose that the winners in two subsets of alternatives have some common alternatives. Then the rule is concordant if these common alternatives are also among the winners in the union of the two subsets. The properties of Chernoff and concordance can be associated with both individual and collective choice functions. In social welfare function the same formal properties are imposed on both individual and collective preference relations. We can make the same requirement for choice-set aggregation rules, i.e. insist that Chernoff and concordance be satisfied by both functions.
Using Aizerman and Aleskerov's example we can show that two quite natural looking aggregation rules fail on either one of these two requirements. Rule 1 is a version of majority rule: whenever an alternative is included in the choice sets of a majority of voters, it will be elected. Rule 2 is plurality: whichever alternative is included in more numerous choice sets than the other alternatives, is elected. The former rule is called local since the inclusion of an alternative in the collective choice set can be determined independently of other alternatives ("locally"). Rule 2, on the other hand, is not local as the determination of whether x belongs to the collective choice set requires comparison with all other alternatives ("globally"). Suppose we have alternatives x, y and z, 3 individuals and the individual choices as indicated in the following table.

alt. set	**ind. choice sets**			**rule 1**	**rule 2**
	ind.1	**ind. 2**	**ind. 3**		
{x,y,z}	{x}	{z}	{y}	empty	{x,y,z}
{x,y}	{x}	{x}	{y}	{x}	{x}
{x,z}	{x}	{z}	{x}	{x}	{x}
{y,z}	{y}	{z}	{y}	{y}	{y}

Clearly concordance is not satisfied by rule 1, since x belongs to the choice sets from {x,y} and {x,z}, but is not included in the choice set from {x,y,z}. On the other hand, rule 2 fails on Chernoff since {z} is included in the choice set from {x,y,z}, but not to one from {x,z}. It is also worth noticing that plurality does not satisfy choice-set monotonicity, while majority does.
The results of Aizerman and Aleskerov are largely negative as far as local aggregation functions are concerned: they typically fail on some rationality conditions. Much less is known about non-local aggregation operators.

Systems Based on Richer Information

Individual choice functions can be regarded as less demanding for the voters than ordinal preference rankings over alternatives. However, we often encounter situations where we in fact have much more preference information about alternatives than just their ranking. Typical examples are situations where the individual's opinions are reflected in her willingness to pay for various alternatives. Since monetary sums are ratio scale variables, the individual is able to signal her preferences in much richer way than in the standard social choice setting.
An relatively recent proposal for a voting system that utilizes richer than ordinal ranking information has recently been made by Balinski and Laraki (2007). The authors call it majority judgment. The procedure is the following. Given a set of alternatives or candidates, the voters evaluate each of them by assigning them a value from a set of values, such as integers from 0 to 10 or from the set {excellent, very good, good, satisfactory, tolerable, poor, to be rejected} or from {A, …F}. The highest grade given by an absolute majority of voters is called the majority grade of the alternative. This is a well-defined concept when the values in the set can be ordered from best to worst. For each alternative there is a value so that more than 50% of the voters give it or higher value to the alternative in question. Now, the majority judgment winner is the alternative with the highest majority grade. If the voters are listed according to the grades they give to a candidate from the lowest to the highest, the majority grade is the one given by the median voter, i.e. the voter with as many other voters on her "lower" and "higher" side. In case the median is not unique, the majority grade is the lowest of the values defin-

[4] Aizerman and Aleskerov call this the heritage condition.

ing the median interval of grades. Ties between candidates are broken in the following manner. For each candidate with the same majority grade, say "good", one tallies the number of voters who give the candidate a higher grade than "good". Let them be b in number. Similarly, one counts the number of voters who give the candidate a grade worse than "good". Say their number is w. If b > w, one gives the candidate the majority grade "good+". If b<w, then the majority grade is "good-". This may help in breaking some ties, but it is still possible that more than one candidate gets the same majority grade, say "good+". In this case the tie is broken by the observing the number of voters who give the candidate a grade higher than "good". The candidate for whom this number is higher wins the tie. In similar vein, for ties involving candidates with "good-" grade, one counts the number of voters with grade lower than "good" assigned to the candidate. The candidate for whom this number is smaller wins the tie.

Balinski and Laraki argue that majoritarian judgment encourages voters to reveal their true valuations of candidates. This is debatable, however. Suppose that the voters have cast their grades to all candidates and that there is a tie for winner between candidates A and B, both of whom get the majority grade "good+" . This means that exactly the same number of voters regard both A and B better than "good". Consider now a voter who grades A as "excellent" and B as "very good". If this voter new the distribution of grades, it would make sense for her to grade B as "tolerable" or something else below "good". Thereby she would break the tie, ceteris paribus, in favour of her favourite A. This shows that in the majority judgement system the sincere revelation of grades does not always lead to a Nash equilibrium. Hence the system is manipulable.

A related system, called utilitarian voting, has been proposed by Hillinger (2004; 2005). It is basically identical with the range voting (Smith 2000). The underlying idea is that for each candidate the voters vote by expressing their cardinal utility values for that candidate. The winner is the candidate with the largest sum of expressed utilities. The first crucial feature of utilitarian voting is that each voter is allowed to assign any grade or score to any alternative, i.e. the score given candidate A in no way restricts the score that can be given to candidate B. The second feature is that the scores are values on a predetermined scale of values, say (0,1) as in approval voting. The third defining characteristic of utilitarian voting is that the winner or, as the case may be, the order of priority among candidates is determined by the sum of scores received from the voters. The candidate with the largest score sum wins.

Obviously the voter input in utilitarian voting is very similar to the one resorted in majority judgment. The method for determining the winner, however, differs. The utilitarian voting elects the candidate with largest average score, while the majority judgment ends up with one associated with the highest median grade. As we just saw, the majoritarian judgment is manipulable. The same example can be used to show that utilitarian voting is manipulable as well. In fact, manipulating these systems is not much different from manipulating the Borda Count. If one knows the toughest contestant of one's favourite, then giving the former the lowest possible grade – regardless of one's true valuation – helps in electing one's favourite. Moreover, in those cases where one's favourite does not get the highest possible grade in one's true evaluation, giving it the maximum grade will increase its likelihood of being elected.

The main point is, however, that alternatives to systems that operate on individual preference rankings exist and deserve scholarly attention. Strategic behaviour in opinion revelation cannot be excluded, but this is a problem in aggregating ordinal preference information as well.

How to Evaluate Systems?

Given the abundance of voting systems and performance criteria as well as the theoretical results on compatibilities between various desiderata, it is worth stopping for a moment to consider what use can we make out of all this information in the design of voting systems. The most straight-forward way to proceed is to use a single criterion of performance to eliminate all those systems failing on this criterion, then possibly pick another criterion to eliminate some of the remaining ones etc. Of course, the criteria used have to be deemed the most important ones. Candidates for this type of elimination criteria are absolute majority or strong Condorcet criterion (if an alternative is ranked first by more than half of the electorate, this alternative has to be elected), the Condorcet winner criterion, monotonicity, consistency, the Condorcet loser criterion (if a candidate would be defeated by all the others in pair-wise majority comparisons, it ought not to be elected), Pareto criterion etc.

There are problems with this type of approach. To wit, which are the most important criteria? The scholarly community is divided on this issue. Even if unanimity prevailed on the most important criteria, this approach would lead to a dichotomy in each criterion: those systems that satisfy the criterion and those which do not. Both classes are bound to consist of many systems.

Within each class one could resort to probability or simulation modelling to find out the theoretical probability or frequency of criterion violations under various "cultures", i.e. assumptions concerning the distribution of preferences in the electorates (Gehrlein 2006). But which culture is most appropriate for this kind of assessment? The early simulations of voting procedures as well as analytical probability models were based on the "impartial culture" assumption according to which the preference ranking for each voter is generated randomly and independently of other voters. It has turned out that this assumption may lead to flawed conclusions about the frequency of various anomalies in voting systems (Regenwetter et al. 2006).[5]

Alternatively, one could utilize multiple performance criteria simultaneously and construct a dominance relation over voting systems. A system dominates another if it satisfies all the criteria – among those considered – that the latter satisfies and at least one criterion that the latter does not satisfy. Once this relation has been constructed, it is natural to make the final choice from among undominated systems. This approach to system choice is not without problems, either. The set of undominated systems is typically pretty large one vis-à-vis the original set of systems. Moreover, the dominance implicitly treats all criteria equally, i.e. they are all deemed of equal importance.

Historically, the choice of the best voting system has been deemed as context-dependent. What has been found right in electing the president has typically not been viewed appropriate in judgment aggregation. Perhaps this is how it should be. Some choice settings emphasize the consensual nature of the outcomes, while in others intensive support of large voter groups seems like a plausible desideratum. In both types of settings a wide variety of systems is available.

The property of manipulability or strategic misrepresentation of opinions is a kind of meta-criterion since it has implications to other system properties. If the system is manipulable and has a set of desirable properties when the sincere opinion revelation assumption is made, it is not guaranteed that those desirable properties also hold when the voters resort to strategic behaviour. Strictly speaking what manipulability entails is that there is a situation where a voter can benefit from misrepresenting her opinion. One such situation is enough to classify a system manipulable. So, a more refined analysis would call for estimates regarding the relative frequency of such situations. In practice, the manipulability of a system depends on at least three different aspects (Nurmi 2002, 110-111):

1. The empirical frequency of those profiles in which an individual or group can benefit from opinion misrepresentation.
2. The nature of information that the voters need in order to misrepresent their opinions with success.
3. The payoff difference that a successful misrepresentation brings about to the voters engaged in it.

So, instead of manipulability as a dichotomous notion one should talk about degrees of manipulability if these three aspects are anything to go by. Kelly's (1993) measure of degree of manipulability focuses precisely on the first aspect by defining the degree of manipulability as the number of profiles in which the procedure is manipulable. This measure can be modified by weighting the profiles by the number of voters who can benefit from misrepresentation (Smith 1999). Nonetheless, one ends up with a measure that is relative to the number of voters and alternatives.

The second aspect is of great practical importance, since the manipulability of a system in principle means very little in practice if the information that the voters need to benefit from opinion misrepresentation is of the kind that they cannot typically possess. On intuitive grounds one could argue that the plurality voting requires no more information about other voters' views than the distribution of the first ranked alternatives, while STV requires a lot more detailed information in order for the misrepresentation to succeed. The third aspects relates directly to the incentives of the voters to misrepresent their opinions: the larger the benefit, the more likely is misrepresentation, ceteris paribus.

As was stated above, the manipulability of a system means that sincere voting strategies do not always lead to Nash equilibrium outcomes. But what does it mean that a voting outcome is not an equilibrium? By definition it means that at least some voters may regret their voting strategies in the sense that *assuming that the others stick to their strategies* they could have brought about a better outcome for themselves had they selected some other voting strategy. But what is there to justify the assumption that the others would not change their behaviour?

[5] This does not make impartial culture simulations worthless. On the contrary they are very useful is assessing the purely procedure-related effects such as how close the winner intuitions of various systems are to each other.

Very little. The notion of equilibrium is based on a counterfactual proposition. This is perhaps worth taking into account in assessing the practical implications of manipulability results.

Searching for consensus

Often voting systems are resorted in an effort to reach a consensus on an issue where several alternative positions are available. The task is trivial when all voters have an identical position on the issue at hand. But in general no such unanimity exists, but has to be found using some procedure. Given a non-unanimous profile of opinions, a plausible way to proceed is to look for the collective opinion that is closest to the expressed opinions of voters. If all but one voter in a large group have an identical preference ranking and the collective choice is to be a ranking as well, the collective ranking closest to the observed one would seem to be the one representing the vast majority opinion. In large bodies the same suggestion would hold when all but very few voters have an identical opinion.

Although pretty obvious in these cases, the search for the nearest collective ranking in general needs some explication. There are many ways of measuring the proximity of two preference rankings. Perhaps the best-known is the inversion metric which tallies the number of individual binary preference inversions needed to transform one ranking into the other. Kemeny's rule is based on this metric (Kemeny 1959). For any given profile of complete and transitive preference orders over k alternatives it determines the closest collective preference ranking by computing for each possible k! ranking its distance (in the sense of the inversion metric) to each individual's ranking and summing these. The collective ranking for which this sum is at the minimum is the Kemeny ranking and its first-ranked alternative the Kemeny winner.[6] Kemeny's rule can, thus, be seen as a method that defines a consensus state and a metric for measuring distances from observed profiles to ranking candidates and that, moreover, results in a ranking for which the sum of distances in at the minimum.

In Kemeny's rule the consensus pertains to every rank in the profile. In many settings the consensus on every position in the collective ranking is something of a luxury. Especially, if the task is elect just one alternative, it makes little sense to search for such a comprehensive consensus. Instead one could look for the collective ranking which is closest to the observed individual rankings and which differs from the individual rankings only in positioning the same alternative first in the ranking. In other words, the collective ranking would be obtained by counting the number of inversions needed to put a given alternative first in every individual ranking and summing those inversions over voters. The winner is then the alternative that needs the smallest number of inversions to end up first in every voter's ranking. Nitzan (1981) shows that the ranking resulting from this minimization is the same as the Borda ranking, i.e. the order based on Borda scores of alternatives.

It turns out that all ranking-based voting systems can be characterized in terms of a goal (consensus) state and a metric used in measuring the distance between the observed preference ranking and the goal state (Meskanen and Nurmi 2006). For example the plurality voting can be defined as the rule which minimizes the distance between the observed profile and a state where all voters have the same alternative ranked first keeping the rankings between the other alternatives intact. Since the goal is the same as in the Borda Count, the metric must be different from the inversion one. Otherwise we would be dealing with the Borda Count. Indeed, the metric for plurality voting is defined so that whenever two rankings differ in terms of the first ranked alternative, their distance is equal to unity. Otherwise, it is zero.

The goal state cum distance metric characterization opens a new angle to analyzing voting systems. It is the decision setting that often determines the most appropriate goal state. Thereafter, the metric captures our views of closeness of different opinions. As these two aspects pin down a voting system, a way to choose the best voting system for any given purpose is to spell out one's intuitions in terms of them, i.e. state explicitly what is the desired goals state and how one measures distances between various views.

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[6] Young (1988) argues that this method is in fact the one that Condorcet had in mind in late 18'th century. Young (1995) shows that Kemeny's method can be seen as resulting in the maximum likelihood ranking.

Conclusion

Theory of voting is approached above from three perspectives: (1) by determining which desirable or undesirable properties various systems possess, (2) by studying the mutual compatibility or incompatibility of the properties, and (3) by characterizing them in terms of various conditions or goal state-metric combinations. The first approach takes its motivation from the fact that the institutional (voting system) design does not take place in a vacuum, but in the setting where historical and cultural features dictate the set of realistic systems within which the choice has to be made. Hence it is important to know the properties – desirable and undesirable – of the systems deemed realistic. While informative and potentially useful, this approach is on par with classification of objects in the theory of measurement, i.e. necessary but only the first step in the way of measuring properties. The second approach is a more advanced in abstracting away concrete voting systems and dealing with the relationships that obtain between their properties. This approach is notorious for its primarily negative results showing the incompatibility of various desiderata of systems. The third approach is either “axiomatic” in the sense of aiming at a characterization of systems with the aid of properties necessary and sufficient for them or distance-based in the sense of determining the underlying goal states and distance metric for each voting system.

Most results in the theory of voting have been achieved under the standard assumption that the voters possess complete and transitive preference relations over the alternatives and that we are looking for optimal ways of aggregating those preferences either into a set of best alternatives or a collective preference ranking. Since many incompatibility results depend on these assumptions, it is worthwhile to look for plausible alternatives for them. The standard assumption can be either too demanding – the voters do not necessarily have preference rankings over all alternatives – or too modest – the voters may have a more refined opinion about the alternatives than a mere ranking. Both of these possibilities have been briefly discussed above. It seems that the emphasis on voting systems studies has recently moved towards aggregating more detailed voter input than preference rankings.

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